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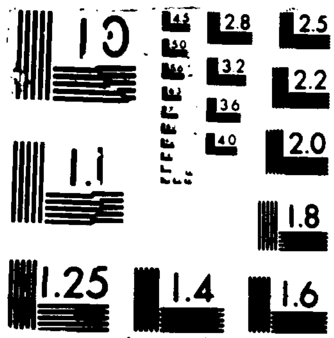
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BIAS FUNCTION OF THE MAXIMUM LIKELIHOOD ESTIMATE OF ABILITY FOR DISCRETE ITEM RESPONSES

AD-A183 811

FUMIKO SAMEJIMA

UNIVERSITY OF TENNESSEE

KNOXVILLE, TENN. 37996-0900

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The paper is concerned with the bias of the maximum likelihood estimate as a function of the latent trait, or ability, for the general case in which item responses are discrete. The rationale is presented, and observations are made with respect to the effects of the test information function, the item parameters, the number of items, the transformation of the latent variable, etc., on the amount of bias.

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The research was conducted at the principal investigator's laboratory, 405 Austin Peay Bldg., Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked as assistants for this research include David M. Immel, Richard D. Strouse, Christine A. Golik, J. Douglas Jenkins, Stacey M. Miller, Sherry C. Campbell, and Robert S. Hiers.

1 Introduction

Lord has proposed and discussed a bias function of the maximum likelihood estimate in the context of the three-parameter logistic model (cf. Lord, 1983). In so doing, he used Taylor's expansion of the likelihood equation and proceeded from there, obtained an equation which includes the conditional expectation of the discrepancy between the maximum likelihood estimate and the true ability, and ignored all terms of orders higher than n^{-1}, where n indicated the number of items.

Let θ be ability, or latent trait, which assumes any real number. Let g $(= 1, 2, \ldots, n)$ denote an item, k_g be a discrete response to item g, and $P_{k_g}(\theta)$ denote the operating characteristic of the discrete response k_g, or the conditional probability, given θ, with which the examinee responds to item g with k_g. The item response information function, $I_{k_g}(\theta)$, is defined by

$$I_{k_g}(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_{k_g}(\theta) \quad , \tag{1.1}$$

and the item information function $I_g(\theta)$ is the conditional expectation of the item response information function, given θ, so that we can write

$$I_g(\theta) = E[I_{k_g}(\theta) \mid \theta] = \sum_{k_g} I_{k_g}(\theta) P_{k_g}(\theta) \quad . \tag{1.2}$$

On the dichotomous response level (Samejima, 1972), the set of operating characteristics for item g is represented by a single function, i.e., the operating characteristic of the positive response, which is called the item characteristic function, or item response function[1].

Let $P_g(\theta)$ be the item characteristic function in the three-parameter logistic model, which is given by,

$$P_g(\theta) = c_g + (1 - c_g)[1 + exp\{-Da_g(\theta - b_g)\}]^{-1} \quad , \tag{1.3}$$

where a_g, b_g and c_g are the item discrimination, difficulty and guessing parameters, and D is a scaling factor, which is set equal to 1.7 when the logistic model is used as a substitute for the normal ogive model. Lord's bias function $B(\theta)$ can be written as

$$B(\theta) = D[I(\theta)]^{-2} \sum_{g=1}^{n} a_g I_g(\theta)[\Psi_g(\theta) - (1/2)] \quad , \tag{1.4}$$

where

$$\Psi_g(\theta) = [1 + exp\{-Da_g(\theta - b_g)\}]^{-1} \quad , \tag{1.5}$$

and $I_g(\theta)$ and $I(\theta)$ are the item information function and the test information function, respectively, which can be given by

$$I_g(\theta) = [P_g'(\theta)]^2 [P_g(\theta)\{1 - P_g(\theta)\}]^{-1} \quad , \tag{1.6}$$

[1] The term, item response function, has been widely used in recent years by researchers who deal solely with the dichotomous response level. From the more comprehensive standpoint, however, this term is ambiguous and misleading, and not appropriate to use. On the graded response level, for example, there may be much more than two item response categories, or there may even be an infinite number of response categories, and the use of item response function for one of these many response categories is not justifiable. For this reason, throughout this paper, the original term, item characteristic function, will be used to indicate the conditional probability for the positive response, given latent trait, on the dichotomous response level.

and

$$I(\theta) = \sum_{g=1}^{n} I_g(\theta) \quad , \tag{1.7}$$

with $P_g'(\theta)$ indicating the first derivative of $P_g(\theta)$ with respect to θ . The former of these two formulae can be given as a special case of the item information function given by (1.2), which is defined for the general case of discrete responses. (Incidentally, in Lord's paper, $B_1(\hat{\theta})$ is used for this bias function. This is not appropriate, however, since it is a function of θ itself, not of its maximum likelihood estimate $\hat{\theta}$.)

2 Rationale

A similar logic can be adopted for the general case, in which item responses are simply discrete. We assume that there are a finite or an enumerable number of k_g 's as possible responses to item g . Thus for the set of n items, we can write for the response pattern V

$$V' = (k_1, k_2, \ldots, k_g, \ldots, k_n) \quad . \tag{2.1}$$

We assume that the operating characteristic $P_{k_g}(\theta)$ is, at least, three-times differentiable with respect to θ . By virtue of local independence, we can write for the likelihood function

$$L_V(\theta) = P_V(\theta) = \prod_{k_g \in V} P_{k_g}(\theta) \quad . \tag{2.2}$$

Thus the likelihood equation is given by

$$\frac{\partial}{\partial\theta} \log L_V(\theta) = \sum_{k_g \in V} \frac{\partial}{\partial\theta} \log P_{k_g}(\theta) \equiv 0 \quad . \tag{2.3}$$

We define $\Gamma_{sk_g}(\theta)$ such that

$$\Gamma_{sk_g}(\theta) = \frac{\partial^s}{\partial\theta^s} \log P_{k_g}(\theta) \tag{2.4}$$

for $s = 1, 2, \ldots$. We notice, in particular, that

$$\Gamma_{1k_g}(\theta) = P_{k_g}'(\theta)[P_{k_g}(\theta)]^{-1} = A_{k_g}(\theta) \quad , \tag{2.5}$$

where $A_{k_g}(\theta)$ is the basic function (Samejima, 1969), and

$$\Gamma_{2k_g}(\theta) = P_{k_g}''(\theta)[P_{k_g}(\theta)]^{-1} - [A_{k_g}(\theta)]^2 \tag{2.6}$$

and

$$\Gamma_{3k_g}(\theta) = P_{k_g}'''(\theta)[P_{k_g}(\theta)]^{-1} - 3A_{k_g}(\theta)P_{k_g}''(\theta)[P_{k_g}(\theta)]^{-1} + 2[A_{k_g}(\theta)]^3 \quad , \tag{2.7}$$

where the superscripts $'$, $''$ and $'''$ indicate the first, second and third partial derivatives of the function with respect to θ , respectively. Thus from (2.3) and (2.5) we can write

$$\sum_{k_g \in V} \Gamma_{1k_g}(\hat{\theta}_V) = \sum_{k_g \in V} A_{k_g}(\hat{\theta}_V) = 0 \quad . \tag{2.8}$$

Let $\Gamma_{sV}(\theta)$ be defined by

$$\Gamma_{sV}(\theta) = \sum_{k_g \in V} \Gamma_{sk_g}(\theta) \tag{2.9}$$

for $s = 1, 2, \ldots$. For a fixed value of θ we can write by Taylor's formula

$$\begin{aligned} \Gamma_{1V}(\hat{\theta}_V) = \Gamma_{1V}(\theta) + (\hat{\theta}_V - \theta)\Gamma_{2V}(\theta) + (1/2)(\hat{\theta}_V - \theta)^2\Gamma_{3V}(\theta) \\ + (1/6)(\hat{\theta}_V - \theta)^3\Gamma_{4V}(\theta) + (1/24)(\hat{\theta}_V - \theta)^4\Gamma_{5V}(\xi) = 0 \end{aligned} \tag{2.10}$$

where ξ is some value between θ and $\hat{\theta}_V$.

Since we have

$$\sum_{k_g} P_{k_g}(\theta) = 1 \quad , \tag{2.11}$$

we obtain

$$\sum_{k_g} \frac{\partial^s}{\partial \theta^s} P_{k_g}(\theta) = 0 \tag{2.12}$$

for $s = 1, 2, \ldots$. Equation (2.12) will be helpful in following the mathematical derivations which are needed in obtaining the bias function. The response pattern information function, $I_V(\theta)$, is defined by

$$I_V(\theta) = -\frac{\partial^2}{\partial \theta^2} \log P_V(\theta) = \sum_{k_g \in V} I_{k_g}(\theta) \quad , \tag{2.13}$$

and the test information function $I(\theta)$ is the conditional expectation of $I_V(\theta)$, given θ , for which we can write

$$I(\theta) = E[I_V(\theta) \mid \theta] = \sum_V I_V(\theta) P_V(\theta) \quad . \tag{2.14}$$

Let $f_{k_g}(\theta)$ be any function of θ defined for a specific discrete response k_g . We can write

$$\begin{aligned} \sum_V \sum_{k_g \in V} f_{k_g}(\theta) P_V(\theta) &= \sum_V \sum_{k_g \in V} f_{k_g}(\theta) P_{k_g}(\theta) P_{V_{-g}}(\theta) \\ &= \sum_{g=1}^{n} \sum_{k_g} f_{k_g}(\theta) P_{k_g}(\theta) \quad , \end{aligned} \tag{2.15}$$

by virtue of the fact that

$$\sum_{V_{-g}} P_{V_{-g}}(\theta) = 1 \tag{2.16}$$

where V_{-g} is the response pattern of $(n-1)$ discrete item scores obtained by deleting k_g from V. Replacing $f_{k_g}(\theta)$ by $I_{k_g}(\theta)$ in (2.15) and using this result, (1.2), (2.13) and (2.14), we can obtain the same equation as (1.7).

Let $\gamma_{sg}(\theta)$ be the conditional expectation of $\Gamma_{sk_g}(\theta)$, given θ, which can be written as

$$\gamma_{sg}(\theta) = E[\Gamma_{sk_g}(\theta) \mid \theta] = \sum_{k_g} \Gamma_{sk_g}(\theta) P_{k_g}(\theta) \quad . \tag{2.17}$$

In particular, we have from (2.5), (2.6), (2.7) and (2.12)

$$\gamma_{1g} = \sum_{k_g} P'_{k_g}(\theta) = 0 \quad , \tag{2.18}$$

$$\gamma_{2g}(\theta) = -\sum_{k_g} [P'_{k_g}(\theta)]^2 P_{k_g}(\theta)^{-1} \tag{2.19}$$

and

$$\gamma_{3g}(\theta) = 2\sum_{k_g} [A_{k_g}(\theta)]^2 P'_{k_g}(\theta) - 3\sum_{k_g} A_{k_g}(\theta) P''_{k_g}(\theta) \quad . \tag{2.20}$$

It is noted from (1.1), (1.2), (2.12) and (2.17) that we can also write

$$\gamma_{2g}(\theta) = -I_g(\theta) \quad . \tag{2.21}$$

We further define $\gamma_s(\theta)$ such that

$$\gamma_s(\theta) = (1/n) \sum_{g=1}^{n} \gamma_{sg} \tag{2.22}$$

for $s = 1, 2, \dots$. In particular, we have

$$\gamma_1(\theta) = 0 \quad , \tag{2.23}$$

$$\gamma_2(\theta) = -(1/n) \sum_{g=1}^{n} I_g(\theta) = -(1/n) I(\theta) \tag{2.24}$$

and

$$\gamma_3(\theta) = (2/n) \sum_{g=1}^{n} \sum_{k_g} [A_{k_g}(\theta)]^2 P'_{k_g}(\theta) - (3/n) \sum_{g=1}^{n} \sum_{k_g} A_{k_g}(\theta) P''_{k_g}(\theta) \quad . \tag{2.25}$$

Let $\epsilon_{sk_g}(\theta)$ and $\epsilon_{sV}(\theta)$ be

$$\epsilon_{sk_g}(\theta) = \Gamma_{sk_g}(\theta) - \gamma_{sg}(\theta) \tag{2.26}$$

and

$$\epsilon_{sV}(\theta) = (1/n) \sum_{k_g \in V} \epsilon_{sk_g}(\theta) \quad , \tag{2.27}$$

respectively. In particular, we can write from these definitions, (1.7), (2.5), (2.6), (2.18) and (2.21)

$$\epsilon_{1k_g}(\theta) = \Gamma_{1k_g}(\theta) = A_{k_g}(\theta) \quad , \tag{2.28}$$

$$\epsilon_{2k_g}(\theta) = P''_{k_g}(\theta)[P_{k_g}(\theta)]^{-1} - [A_{k_g}(\theta)]^2 + I_g(\theta) \quad , \tag{2.29}$$

$$\epsilon_{1V}(\theta) = (1/n) \sum_{k_g \in V} A_{k_g}(\theta) \tag{2.30}$$

and

$$\epsilon_{2V}(\theta) = (1/n) \sum_{k_g \in V} P''_{k_g}(\theta)[P_{k_g}(\theta)]^{-1} - (1/n) \sum_{k_g \in V} [A_{k_g}(\theta)]^2 + (1/n)I(\theta) \quad . \tag{2.31}$$

We can also obtain from (2.15), (2.17), (2.22), (2.26) and (2.27) for the conditional expectation of $\epsilon_{sV}(\theta)$, given θ ,

$$E[\epsilon_{sV}(\theta) \mid \theta] = \sum_{V} \epsilon_{sV}(\theta) P_V(\theta) = \gamma_s(\theta) - \gamma_s(\theta) = 0 \quad . \tag{2.32}$$

With these definitions of $\gamma_s(\theta)$ and $\epsilon_{sV}(\theta)$ and from (2.10) we have

$$\begin{aligned} &\epsilon_{1V}(\theta) + (\hat{\theta}_V - \theta)[\gamma_2(\theta) + \epsilon_{2V}(\theta)] + (1/2)(\hat{\theta}_V - \theta)^2[\gamma_3(\theta) + \epsilon_{3V}(\theta)] \\ &\qquad + (1/6)(\hat{\theta}_V - \theta)^3[\gamma_4(\theta) + \epsilon_{4V}(\theta)] + (1/24)(\hat{\theta}_V - \theta)^4 \Gamma_{5V}(\theta) \doteq 0 \quad , \end{aligned} \tag{2.33}$$

and proceeding from here by taking the conditional expectation of each term in (2.33) with respect to V , given θ , and ignoring all terms whose orders are higher than n^{-1} , we obtain

$$E[\epsilon_{1V}(\theta) \mid \theta] + \gamma_2(\theta) E[\hat{\theta}_V - \theta \mid \theta] + E[(\hat{\theta}_V - \theta)\epsilon_{2V}(\theta) \mid \theta] + (1/2)\gamma_3(\theta) E[(\hat{\theta}_V - \theta)^2 \mid \theta] \doteq 0 \quad . \tag{2.34}$$

It is obvious from (2.32) that the first term on the left hand side of (2.34) disappears. As for the fourth and last term in (2.34), we can use the asymptotic variance of the distribution of the maximum likelihood estimate as the approximation to its last factor, i.e.,

$$E[(\hat{\theta}_V - \theta)^2 \mid \theta] \doteq [I(\theta)]^{-1} \quad . \tag{2.35}$$

Since $\gamma_2(\theta)$ and $\gamma_3(\theta)$ are given by (2.24) and (2.25), respectively, all we need to do is to evaluate the third term on the left hand side of (2.34) in the general framework. In so doing we need to multiply (2.33) by $\epsilon_{2V}(\theta)$, take its expectation with respect to V and ignore all terms of $o(n^{-1})$, to obtain

$$E[(\hat{\theta}_V - \theta)\epsilon_{2V}(\theta) \mid \theta] = -[\gamma_2(\theta)]^{-1} E[\epsilon_{1V}(\theta)\epsilon_{2V}(\theta) \mid \theta] \quad . \tag{2.36}$$

Thus the remaining task is to evaluate the second factor of the right hand side of (2.36). From (2.30) and (2.31) we have

$$
\begin{aligned}
(2.37)\qquad E[\epsilon_{1V}(\theta)\epsilon_{2V}(\theta) \mid \theta] &= \sum_{V} \epsilon_{1V}(\theta)\epsilon_{2V}(\theta)P_V(\theta) \\
&= (1/n^2)\sum_{V}\sum_{k_g \in V} A_{k_g}(\theta) \sum_{k_h \in V} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_V(\theta) \\
&\quad - (1/n^2)\sum_{V}\sum_{k_g \in V} A_{k_g}(\theta) \sum_{k_h \in V} [A_{k_h}(\theta)]^2 P_V(\theta) \\
&\quad + [(1/n)\sum_{g=1}^{n}\sum_{k_g} A_{k_g}(\theta)P'_{k_g}(\theta)]E[\epsilon_{1V}(\theta) \mid \theta] \ .
\end{aligned}
$$

It is obvious from (2.32) that the third term on the right hand side of (2.37) disappears. We can write by virtue of (2.15)

$$
\begin{aligned}
(2.38)\qquad & \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta) \sum_{k_h \in V} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_V(\theta) \\
&= \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta)P''_{k_g}(\theta)[P_{k_g}(\theta)]^{-1}P_V(\theta) \\
&\quad + \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta) \sum_{\substack{k_h \in V \\ h \neq g}} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_V(\theta) \\
&= \sum_{g=1}^{n}\sum_{k_g} A_{k_g}(\theta)P''_{k_g}(\theta) \\
&\quad + \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta)P_{k_g}(\theta) \sum_{\substack{k_h \in V \\ h \neq g}} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_{V_{-g}}(\theta) \ .
\end{aligned}
$$

It is also obvious from (2.5), (2.12) and (2.15) that we can further rewrite the second term of the rightest hand side of (2.38) in such a way that

$$
\begin{aligned}
(2.39)\qquad & \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta)P_{k_g}(\theta) \sum_{\substack{k_h \in V \\ h \neq g}} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_{V_{-g}}(\theta) \\
&= \sum_{g=1}^{n}\sum_{k_g} A_{k_g}(\theta)P_{k_g}(\theta)\sum_{V_{-g}}\sum_{k_h \in V_{-g}} P''_{k_h}(\theta)[P_{k_h}(\theta)]^{-1}P_{v_{-g}}(\theta) \\
&= \sum_{g=1}^{n}\sum_{k_g} P'_{k_g}(\theta)\sum_{h \neq g}\sum_{k_h} P''_{k_h}(\theta) = 0 \ .
\end{aligned}
$$

Following a similar process, we have

$$
\begin{aligned}
(2.40)\qquad \sum_{V}\sum_{k_g \in V} A_{k_g}(\theta) \sum_{k_h \in V} [A_{k_h}(\theta)]^2 P_V(\theta) &= \sum_{V}\sum_{k_g \in V} [A_{k_g}(\theta)]^2 \sum_{k_h \in V} A_{k_h}(\theta)P_V(\theta) \\
&= \sum_{V}\sum_{k_g \in V} [A_{k_g}(\theta)]^3 P_V(\theta)
\end{aligned}
$$

$$+\sum_{V}\sum_{k_g\in V}[A_{k_g}(\theta)]^2\sum_{\substack{k_h\in V\\ h\neq g}}A_{k_h}(\theta)P_V(\theta)$$

$$=\sum_{g=1}^{n}\sum_{k_g}[A_{k_g}(\theta)]^2P'_{k_g}(\theta) \ .$$

Substituting these results into (2.37) and rearranging, we obtain

$$E[\epsilon_{1V}(\theta)\epsilon_{2V}(\theta)\mid\theta]=(1/n^2)\sum_{g=1}^{n}\sum_{k_g}A_{k_g}(\theta)[P''_{k_g}(\theta)-A_{k_g}(\theta)P'_{k_g}(\theta)] \ . \tag{2.41}$$

Thus we can write from this result, (2.24) and (2.36)

$$E[(\hat{\theta}_V-\theta)\epsilon_{2V}(\theta)\mid\theta]=(1/n)[I(\theta)]^{-1}\sum_{g=1}^{n}\sum_{k_g}A_{k_g}(\theta)[P''_{k_g}(\theta)-A_{k_g}(\theta)P'_{k_g}(\theta)] \ , \tag{2.42}$$

where $P'_{k_g}(\theta)$ and $P''_{k_g}(\theta)$ indicate the first and second derivatives of $P_{k_g}(\theta)$ with respect to θ, respectively. Substituting (2.21), (2.22), (2.35) and (2.42) into (2.34) and rearranging, we obtain for the bias function, $B(\theta)$, of the maximum likelihood estimate

$$B(\theta)=E[\hat{\theta}_V-\theta\mid\theta]=-(1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}\sum_{k_g}A_{k_g}(\theta)P''_{k_g}(\theta) \tag{2.43}$$

$$=-(1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}\sum_{k_g}P'_{k_g}(\theta)P''_{k_g}(\theta)[P_{k_g}(\theta)]^{-1} \ .$$

It is obvious from this result that the bias of the maximum likelihood estimate on the discrete response level has the negative relationship with the amount of test information, i.e., we can expect a small amount of bias when the amount of test information is large, and vice versa. The relationship is rather complicated, however, because of the numerator of the rightest hand side of (2.43), which includes $P_{k_g}(\theta)$ and its first and second derivatives with respect to θ.

On the graded response level, where item score x_g assumes successive integers, 0 through m_g, each k_g in (2.43) must be replaced by x_g. On the dichotomous response level, it can be reduced to the form

$$B(\theta)=E[\hat{\theta}_V-\theta\mid\theta]=(-1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}P'_g(\theta)P''_g(\theta)[P_g(\theta)Q_g(\theta)]^{-1} \ , \tag{2.44}$$

where

$$Q_g(\theta)=1-P_g(\theta) \ , \tag{2.45}$$

with $P''_g(\theta)$ indicating the second derivative of $P_g(\theta)$ with respect to θ. When $P'_g(\theta)$ is nonzero throughout the entire range of θ, we can also write

$$B(\theta)=E[\hat{\theta}_V-\theta\mid\theta]=(-1/2)[I(\theta)]^{-2}\sum_{g=1}^{n}I_g(\theta)P''_g(\theta)[P'_g(\theta)]^{-1} \ . \tag{2.46}$$

This includes Lord's bias function in the three-parameter logistic model as a special case.

3 Dichotomous Response Level

On the dichotomous response level where we deal only with two item score categories, as is exemplified by such a pair as "right" and "wrong", or "agree" or "disagree", the most commonly used family of models may be the one in which the item characteristic function is strictly increasing in θ . In such a case, $P_g'(\theta)$, the first derivative of the item characteristic function with respect to θ , is nonnegative. If, in addition, $P_g'(\theta)$ is unimodal, as is the case with many commonly used mathematical models, the second derivative, $P_g''(\theta)$, assumes positive values up to the modal point, and then it has negative values. A close examination of (2.46) reveals that, in such a model, the direction of bias is positive for very high levels of θ , and it is negative for very low levels of θ . In other words, individuals of very high levels of ability tend to be overevaluated, and those of very low levels of ability tend to be underevaluated.

Now we shall observe the bias functions in some specified models which belong to this family.

3.1 Normal Ogive Model

In the normal ogive model, the item characteristic function is given by

$$P_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} e^{-u^2/2}\, du \quad , \tag{3.1}$$

where a_g and b_g are the item discrimination and difficulty parameters, respectively. From (3.1), we can write for the first and second derivatives of $P_g(\theta)$ with respect to θ

$$P_g'(\theta) = a_g (2\pi)^{-1/2} e^{-a_g^2(\theta-b_g)^2/2} \tag{3.2}$$

and

$$P_g''(\theta) = -a_g^2(\theta - b_g) P_g'(\theta) \quad , \tag{3.3}$$

respectively. Substituting (3.2) and (3.3) into (2.46) and rearranging, we obtain for the bias function

$$B(\theta) = (1/2)[I(\theta)]^{-2} \sum_{g=1}^{n} a_g^2(\theta - b_g) I_g(\theta) \quad . \tag{3.4}$$

It is obvious from its definition, which is given by (1.6), that the item information function, $I_g(\theta)$, is nonnegative regardless of the mathematical model. From this, we can see that for a fixed value of θ the sign of the term under the summation sign in (3.4) depends upon the value of the difficulty parameter b_g of each item g . We can also see that, if all the n items have the same values of difficulty parameter, i.e., $b = b_1 = b_2 = \ldots = b_n$, then the bias function is strictly increasing in θ , and equals zero only at $\theta = b$, with positive and negative infinities as its two asymptotes. Moreover, since $P_g(\theta)$ is point-symmetric with $(b, 0.5)$ as the point of symmetry, the bias function is also point-symmetric with the same point of symmetry. In this situation, generally speaking, we should expect a substantial amount of bias as we depart from $\theta = b$.

In many situations of practical importance, however, it is desirable to have a test whose bias function practically assumes zero for a wide range of θ . Equation (3.4) suggests that, in order to materialize such a test, we must develop a set of items whose difficulty parameters distribute widely and evenly, so that, for a wide interval of θ , the negative and positive terms under the summation sign of the right hand side of (3.4) practically "cancel each other out".

Table 3-1 presents the estimated item discrimination parameter $\hat{a}_g$ and item difficulty parameter $\hat{b}_g$ for each of the forty-three dichotomous test items of the Level 11 Vocabulary Subtest of the Iowa Tests of Basic Skills, which were obtained by assuming the normal ogive model (Samejima, 1984a). The data were collected for 2,356 school children of approximately age eleven by the Iowa Testing Bureau, and were analyzed by using the tetrachoric correlation matrix and the principal factor solution of factor analysis. Thus we have for the item parameter estimates

$$\hat{a}_g = \rho_g(1 - \rho_g^2)^{-1/2} \tag{3.5}$$

and

$$\hat{b}_g = \gamma_g \rho_g^{-1} \quad , \tag{3.6}$$

where ρ_g is the factor loading of item g on the single, dominating common factor, which is operationally defined as θ , and γ_g is the normal deviate corresponding to the proportion correct, p_g , of item g . Corresponding results for each of the fifty-five dichotomous test items of Test J1 of Shiba's Word/Phrase Comprehension Tests are also presented as Table 3-2 (cf. Shiba, 1978, Samejima, 1984b). Those data were based upon 2,259 junior high school students in Japan.

Figure 3-1 presents the square root of the test information function of each of these two tests by solid and dashed lines, respectively. We can see that these curves are fairly similar. The bias functions, which were obtained by (3.4) for the two tests, are shown in Figure 3-2. We can see that, over all, the bias is less conspicuous for Test J1 than for Iowa Subtest. In order to show the relationship between the amount of bias and that of test information, Figure 3-3 presents the square root of test information and the bias function together for each of the two tests. It is interesting to note that in both cases, if we tolerate biases of up to ± 0.1 , for example, then the range of θ in which this is the case corresponds to the interval of θ where the square root of test information is approximately 1.75 or greater, or where the amount of test information is approximately 3.0 or greater.

3.2 Logistic Model

Since the logistic model can be considered as a special case of the three-parameter logistic model in which we set the guessing parameter, c_g , equal to zero, Lord's bias function, which is written as formula (1.4) in the present paper, is also applicable. Note, however, that neither $I_g(\theta)$ nor $I(\theta)$ in the formula includes the guessing parameter c_g , when it is used for the logistic model.

A close examination of (1.4) reveals strong similarities of the logistic model with the normal ogive model, i.e., 1) for a fixed value of θ , the sign of the term under the summation sign in (1.4) depends upon the difficulty parameter b_g of each item g ; 2) if $b = b_1 = b_2 = \ldots = b_n$, the bias function is strictly increasing in θ with positive and negative infinities as its two asymptotes, which equals zero only at $\theta = b$, and is point-symmetric with $(b, 0.5)$ as the point of symmetry; and 3) in order to make the bias practically nil for a wide range of θ , we must develop items whose difficulty parameters distribute widely and evenly.

Figures 3-4 and 3-5 present two sets of examples of the square root of the test information function and the bias function obtained by following the logistic model with $D = 1.7$, by solid and dashed lines, respectively. They are the results obtained by using the same set of estimated discrimination and difficulty parameters of the Iowa Level 11 Vocabulary Subtest and Shiba's Test J1, which are shown in Tables 3-1 and 3-2, respectively, as we did for the normal ogive model in the preceding section. As is expected, these results are close to those obtained by following the normal ogive model. As was done in the normal ogive model, the square root of test information and the bias function are put together for each of the two tests, and presented in Figure 3-6. The relationship between the square root of test information and the amount of bias appears to be almost the same as was recognized in the corresponding result obtained by following the normal ogive model, which we discussed in the preceding section.

TABLE 3-1

Estimated Item Discrimination Parameter $\hat{a}_g$ and Item Difficulty Parameter $\hat{b}_g$ for Each of the Forty-Three Dichotomous Test Items of the Level 11 Vocabulary Subtest of the Iowa Tests of Basic Skills, Based upon the Result Collected for 2,356 School Children of Approximately Age Eleven.

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
24	0.196	-4.257
25	0.829	-1.000
26	0.614	-0.821
27	0.594	-0.340
28	0.669	-0.900
29	0.867	-1.077
30	0.956	-0.557
31	0.938	-0.179
32	0.940	-0.803
33	0.434	-2.331
34	0.598	-1.210
35	0.489	-0.569
36	0.657	-0.987
37	0.351	0.577
38	0.665	-0.468
39	0.333	-0.676
40	0.683	0.402
41	0.531	-0.948
42	0.436	0.258
43	0.672	-0.867
44	0.143	4.175
45	0.898	-0.357

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
46	0.612	-0.318
47	0.494	-0.781
48	0.849	0.054
49	0.421	-0.626
50	0.346	-0.250
51	0.664	-0.420
52	0.640	0.217
53	0.402	0.526
54	0.573	0.126
55	0.667	-0.342
56	0.593	1.007
57	0.370	0.398
58	0.416	0.782
59	0.491	-0.731
60	0.678	-0.170
61	0.519	0.748
62	0.938	-0.485
63	0.637	-0.398
64	0.818	-0.042
65	0.606	0.595
66	0.604	-0.376

TABLE 3-2

Estimated Item Discrimination Parameter $\hat{a}_g$ and Item Difficulty Parameter $\hat{b}_g$ for Each of the Fifty-Five Dichotomous Test Items of Test J1 of Shiba's Word/Phrase Comprehension Tests Collected for 2,259 Junior High School Students.

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
J101	0.726	-0.238
J102	0.537	-0.956
J103	0.568	-1.263
J104	0.710	-0.809
J105	0.794	-0.097
J106	0.495	-0.741
J107	0.583	0.205
J108	0.771	-1.974
J109	0.386	-0.872
J110	0.572	-0.327
J111	0.950	-1.266
J112	0.437	-1.036
J113	0.508	-1.061
J114	0.472	0.486
J115	0.704	-0.224
J116	0.303	-1.671
J117	0.390	-0.626
J118	0.583	-1.573
J119	0.653	-0.972
J120	0.293	1.058
J121	0.470	-0.904
J122	0.451	-1.038
J123	0.456	0.151
J124	0.562	-1.313
J125	0.450	-1.691
J126	0.367	-0.424
J127	0.525	-1.299
J128	0.679	-1.094

Item g	Discrimination Parameter $\hat{a}_g$	Difficulty Parameter $\hat{b}_g$
J129	0.761	1.416
J130	0.351	-1.839
J131	0.798	-0.494
J132	0.322	0.162
J133	0.822	-1.377
J134	0.302	1.633
J135	0.850	-0.225
J136	0.368	0.264
J137	0.591	0.331
J138	—	—
J139	0.375	1.602
J140	0.422	0.216
J141	0.566	-0.689
J142	0.447	0.132
J143	0.586	-0.100
J144	0.384	-0.399
J145	0.630	-0.479
J146	0.880	0.057
J147	0.333	0.374
J148	0.521	-0.062
J149	0.509	-0.108
J150	0.512	-0.040
J151	0.462	0.907
J152	0.394	0.478
J153	0.384	2.029
J154	0.242	2.353
J155	0.738	1.258
J156	0.655	1.468

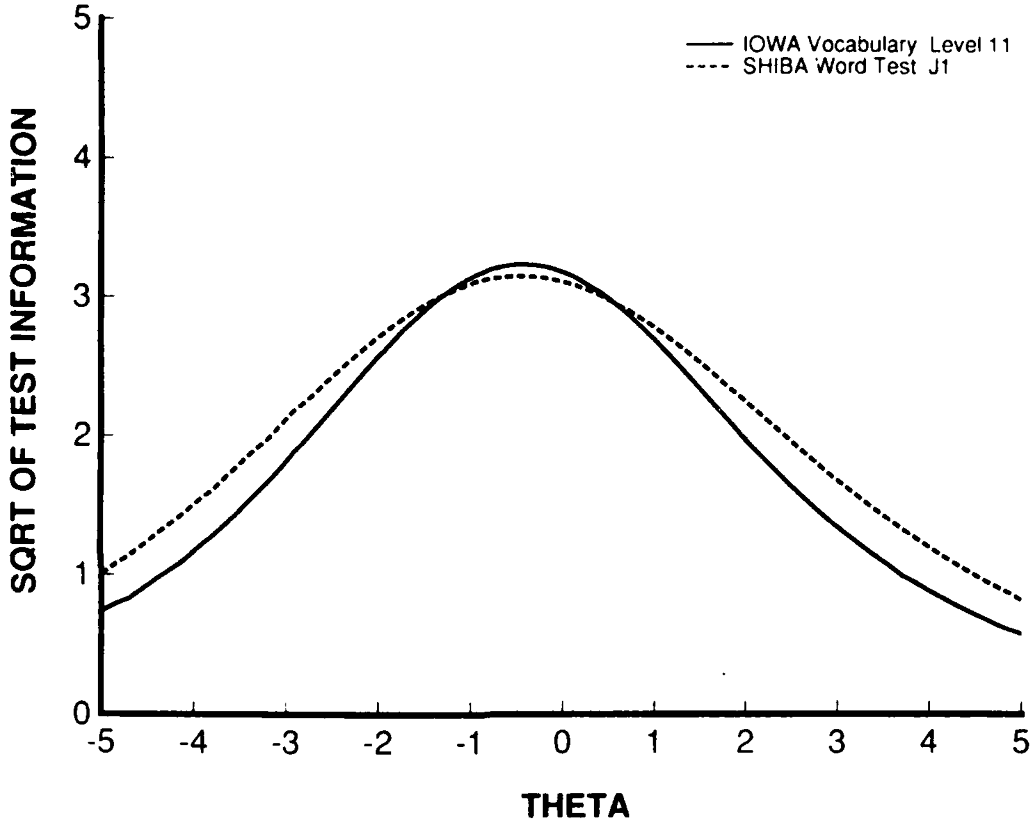


FIGURE 3-1

Square Roots of the Test Information Functions for the Iowa Level 11 Vocabulary Subtest (Solid Line) and for Shiba's Test J1 (Dashed Line), Following the Normal Ogive Model.

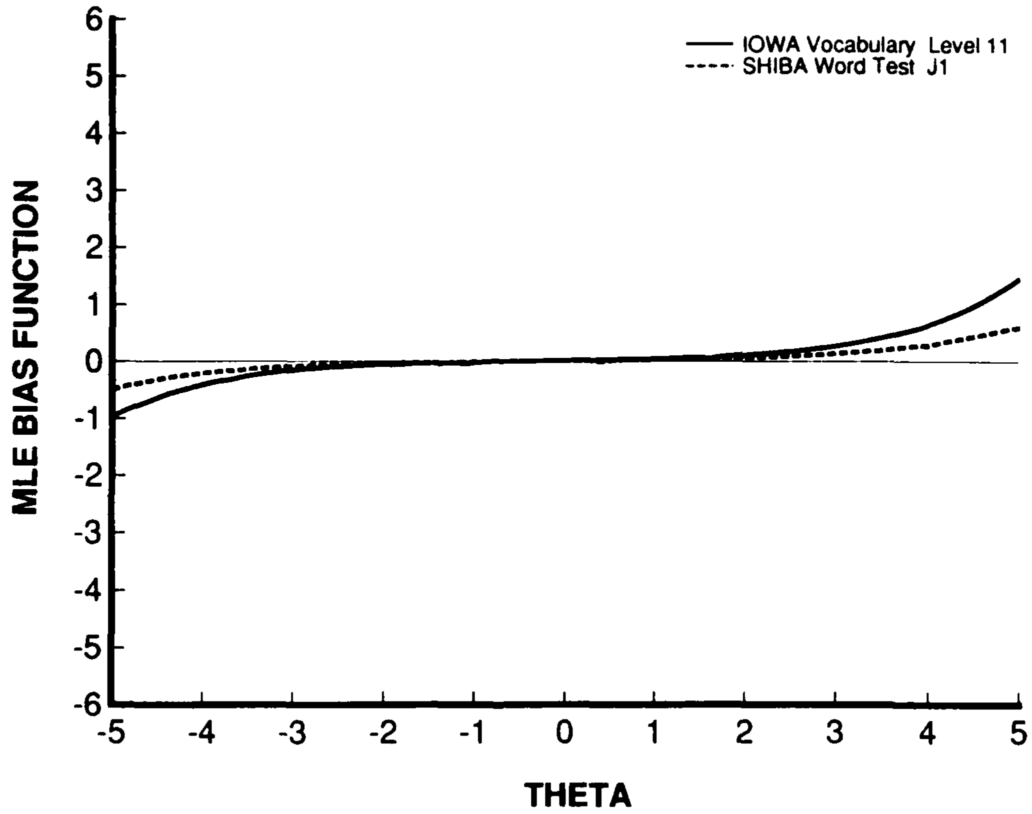
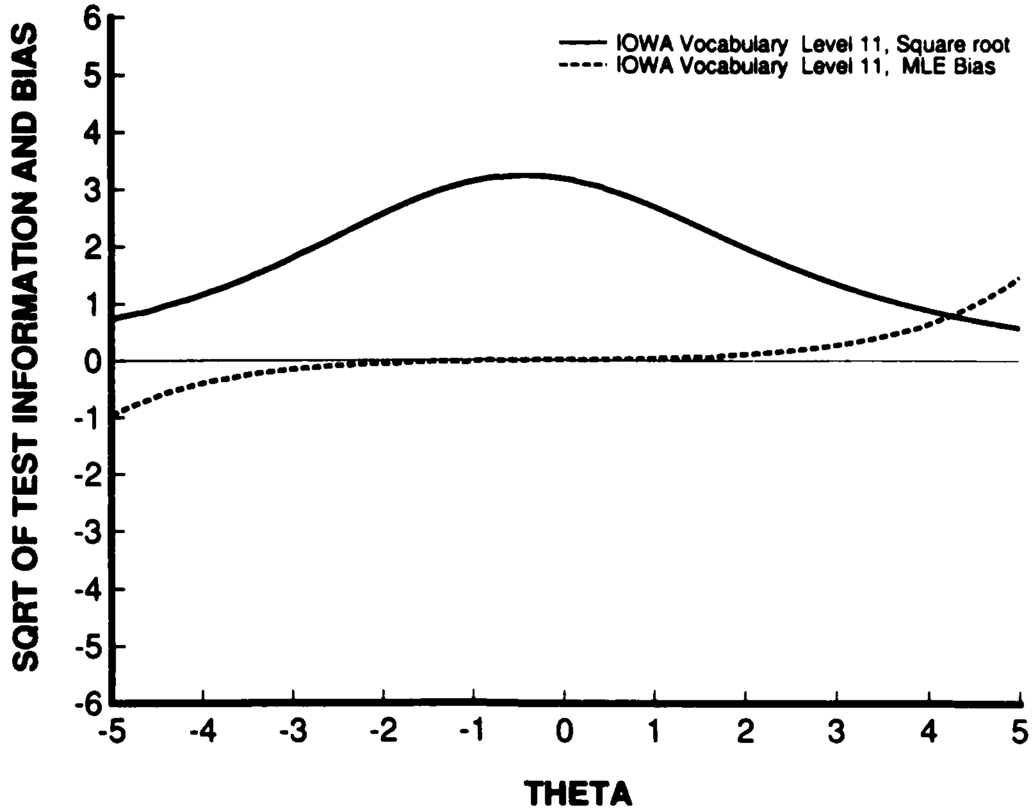


FIGURE 3-2

Bias Functions of the Iowa Level 11 Vocabulary Subtest (Solid Line) and of Shiba's Test J1 (Dashed Line), Following the Normal Ogive Model.



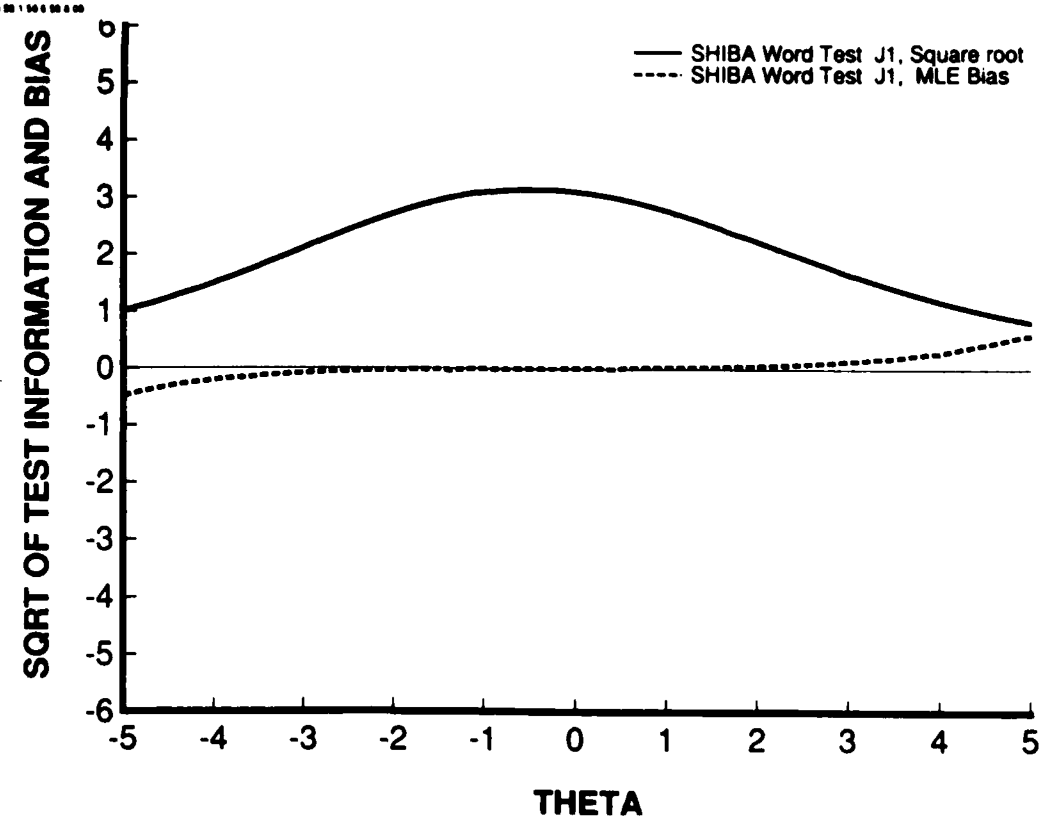


FIGURE 3-3

Comparison of the Square Root of the Test Information Function (Solid Line) and the Bias Function (Dashed Line) Following the Normal Ogive Model, of Each of the Two Tests, i.e., the Iowa Level 11 Vocabulary Subtest (Upper Graph) and Shiba's Test J1 (Lower Graph).

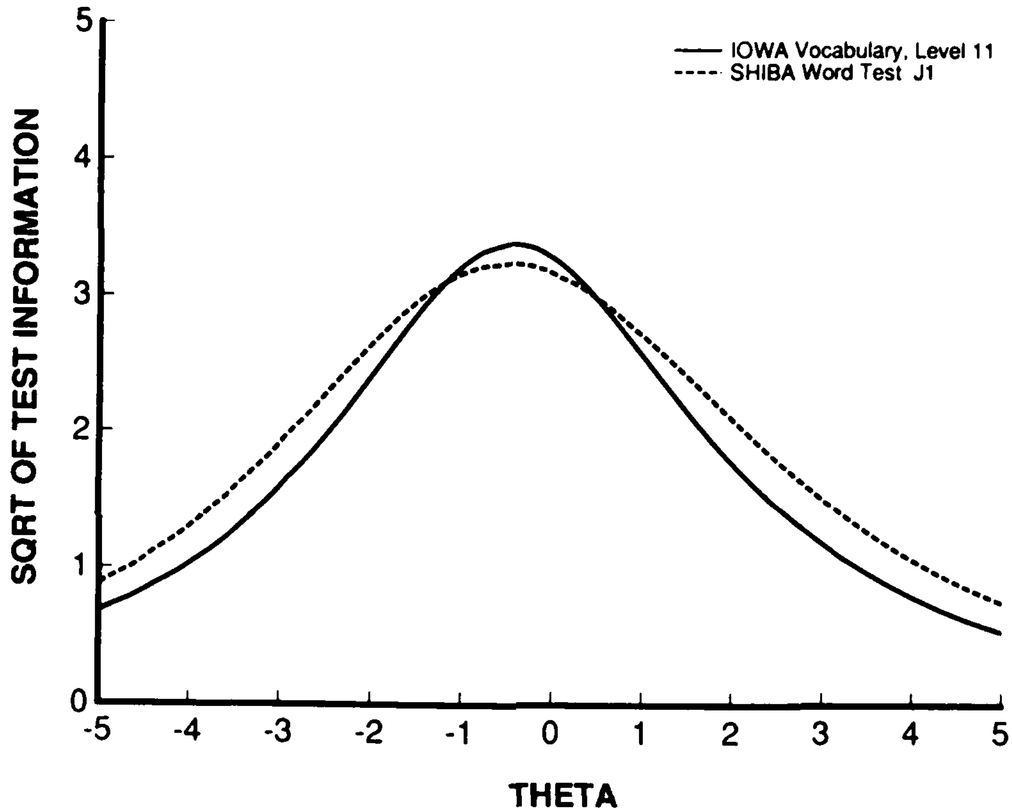


FIGURE 3-4

Square Roots of the Test Information Functions for the Iowa Level 11 Vocabulary Subtest (Solid Line) and for Shiba's Test J1 (Dashed Line), Following the Logistic Model.

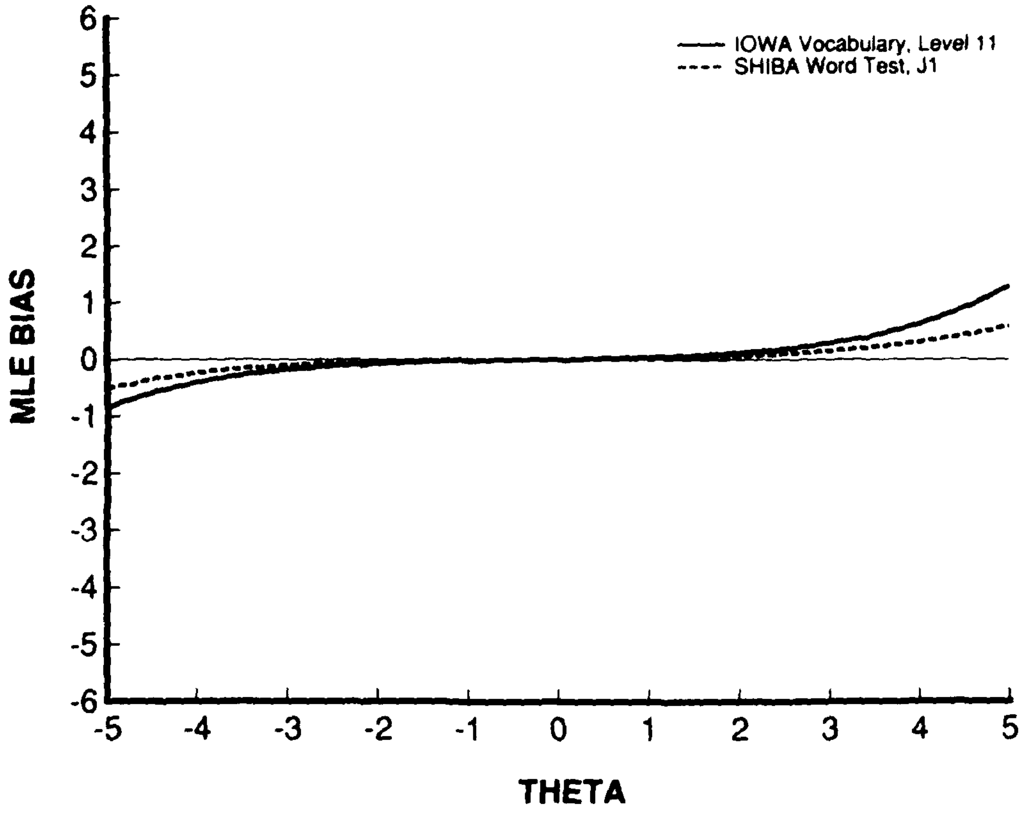
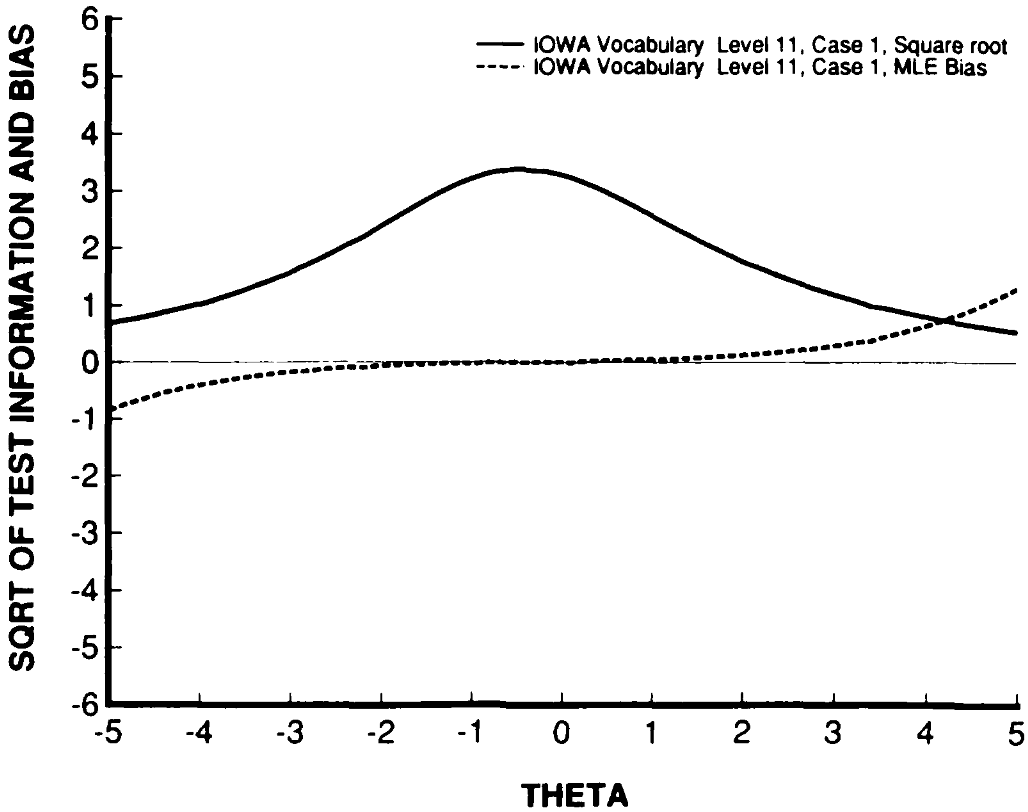


FIGURE 3-5

Bias Functions of the Iowa Level 11 Vocabulary Subtest (Solid Line) and of Shiba's Test J1 (Dashed Line), Following the Logistic Model.



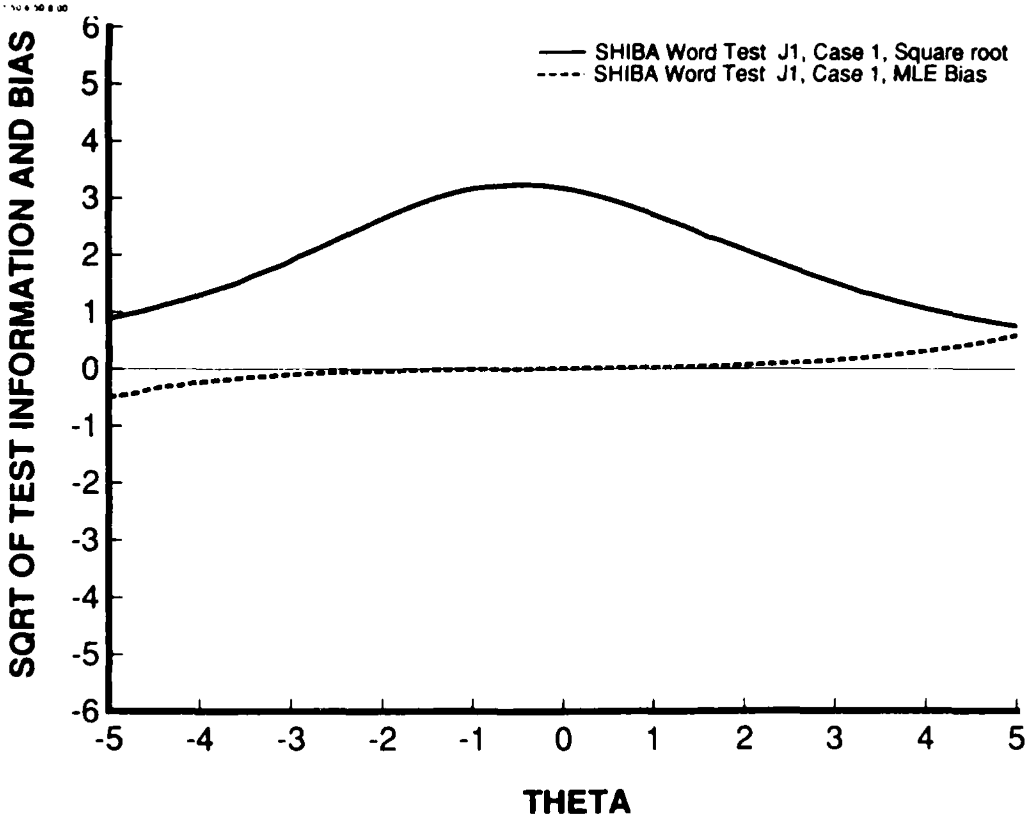


FIGURE 3-6

Comparison of the Square Root of the Test Information Function (Solid Line) and the Bias Function (Dashed Line) Following the Logistic Model, of Each of the Two Tests, i.e., the Iowa Level 11 Vocabulary Subtest (Upper Graph) and Shiba's Test J1 (Lower Graph).

3.3 Rasch Model

Since the Rasch model is a special case of the logistic model, in which the discrimination parameters of all the items are identical, i.e., $a = a_1 = a_2 = \ldots = a_n$, the bias function is obtained by removing a_g on the right hand side of formula (1.4), provided that the scale unit of θ be adjusted to the common discrimination parameter. Thus all the observations made for the logistic model also apply for the Rasch model.

Figure 3-7 presents the item characteristic functions of 25 items, each of which follows the Rasch model with $D = 1.7$. The item difficulty parameters of these items are equally spaced, starting with $b_g = -3.0$ and ending with $b_g = +3.0$ with equal step widths of 0.25 . The square root of the test information of this hypothetical test is shown by a solid line in Figure 3-8. In the same figure, also presented are the square root of test information of each of the two subtests, i.e. the subtest of 13 items which is constructed by taking every other curve in Figure 3-7, and the subtest of 7 items obtained by changing the equal steps from 0.25 to 1.0 . They are drawn by dashed and dotted lines, respectively. The bias functions of these three tests are shown in Figure 3-9 using the same types of lines. In this figure, unlike the square root of test information, it looks as if substantial changes in the number of items did not affect the bias functions to a great extent, especially for the range of θ , -2.0 through 2.0 . In order to show the relationship between the square root of test information and the bias function more clearly, Figure 3-10 presents both curves together for each of the three hypothetical tests. If we tolerate biases of ± 0.1 , as we did before, then for the 25 item test, the critical value of the square root of test information is approximately 2.0 , and for the 13 and 7 item tests, these values are approximately 1.6 and 1.1 , respectively.

This result indicates the importance of the configuration of the item difficulty parameters, i.e., even if the number of items is as small as seven, the approximate unbiasedness can be reached for a wide range of θ , provided that the item difficulty parameters are distributed evenly for a wide range.

3.4 Three-Parameter Logistic Model

In the three-parameter logistic model, the item characteristic function is not point-symmetric, as is clear from the formula (1.3). For this reason, even if the difficulty parameters of all the n items are equal, the bias function, which is given by (1.4), is not point-symmetric either, unlike those in the normal ogive and logistic models. Since random guessing is nothing but noise, there is a certain amount of decrement in the accuracy of estimation, especially on the lower levels of ability or latent trait. Consequently, we must expect a larger amount of bias, especially on the lower levels.

For the purpose of illustration, the guessing parameters, 0.20 and 0.25 , were added to the estimated discrimination and difficulty parameters of each of the 43 test items of the Iowa Level 11 Vocabulary Subtest, which are shown in Table 3-1, respectively, to create two more hypothetical tests. Figure 3-11 presents the square roots of the test information function of these two hypothetical tests by dashed and dotted lines, respectively, in comparison with the one following the (two-parameter) logistic model, which is shown by a solid line. We can see that, in each case, the decrement caused by the guessing parameters is substantial, especially on the lower levels of ability. The bias functions of these two hypothetical tests are shown in Figure 3-12 in comparison with the one for the logistic model, using the corresponding types of lines. It is obvious that random guessing causes a substantial amount of additional bias, especially on the lower levels of ability. Figure 3-13 compares the square root of test information with the amount of bias for each of the two hypothetical tests, as we did previously. It looks as if the same rule held in these two cases of the three-parameter logistic model, i.e., the amount of bias is within the range of ± 0.1 for the intervals of θ for which the square root of test information is approximately 1.75 or greater, just as we observed in the examples of the (two-parameter) normal ogive and logistic models. Such intervals of θ are substantially smaller, however.

In a similar manner, two additional hypothetical tests were created with $c_g = 0.20$ and $c_g = 0.25$ as the guessing parameters, respectively, added to the estimated discrimination and difficulty parameters of each item of Shiba's Test J1, which are shown in Table 3-2. These results are presented in Figures 3-14 through 3-16. We can see in these figures that the results are very similar to the corresponding

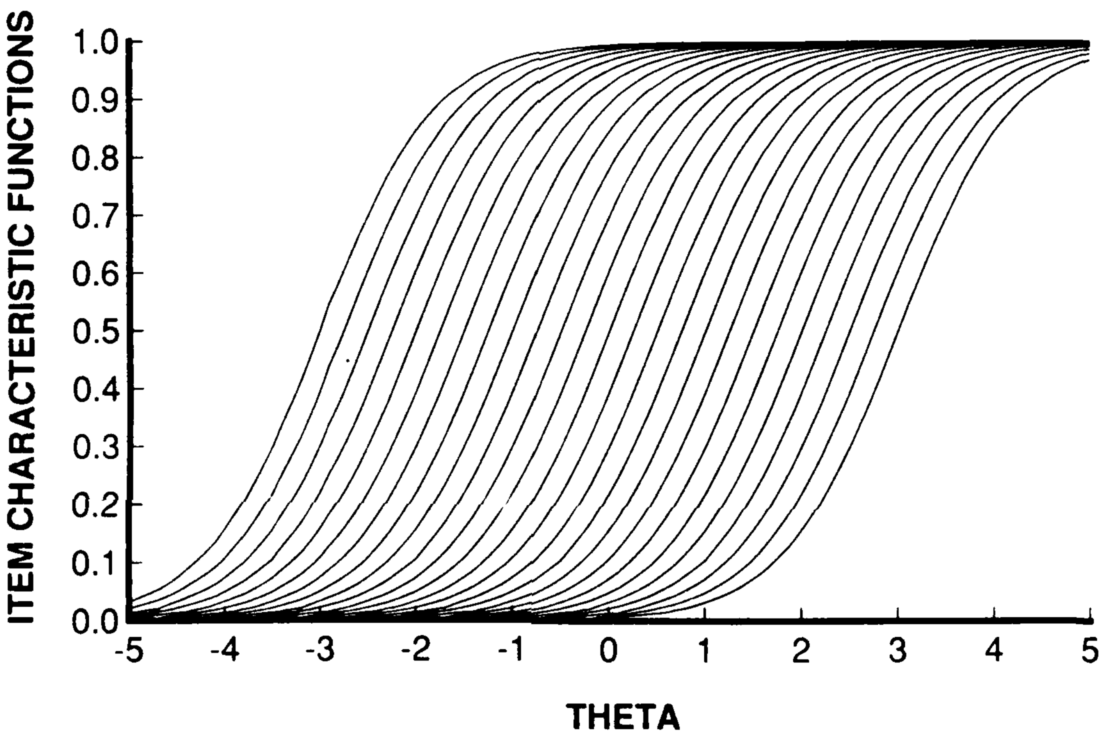


FIGURE 3-7

Twenty-Five Item Characteristic Functions Following Rasch Model with $D = 1.7$ and Equally Spaced Difficulty Parameters Ranging from -3.0 to $+3.0$.

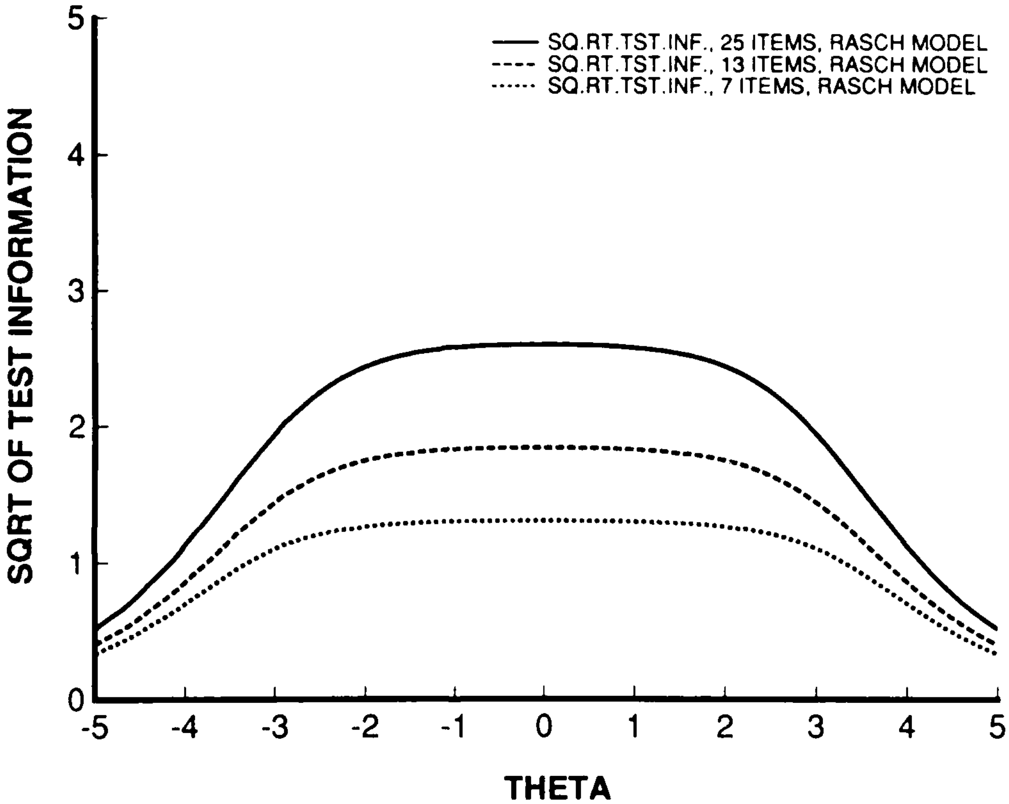


FIGURE 3-8

Square Root of Test Information of Each of Three Hypothetical Tests of 25 (Solid Line), 13 (Dashed Line) and 7 (Dotted Line) Items, Respectively, Following Rasch Model with $D = 1.7$ and Equally Spaced Difficulty Parameters Ranging from -3.0 to $+3.0$.

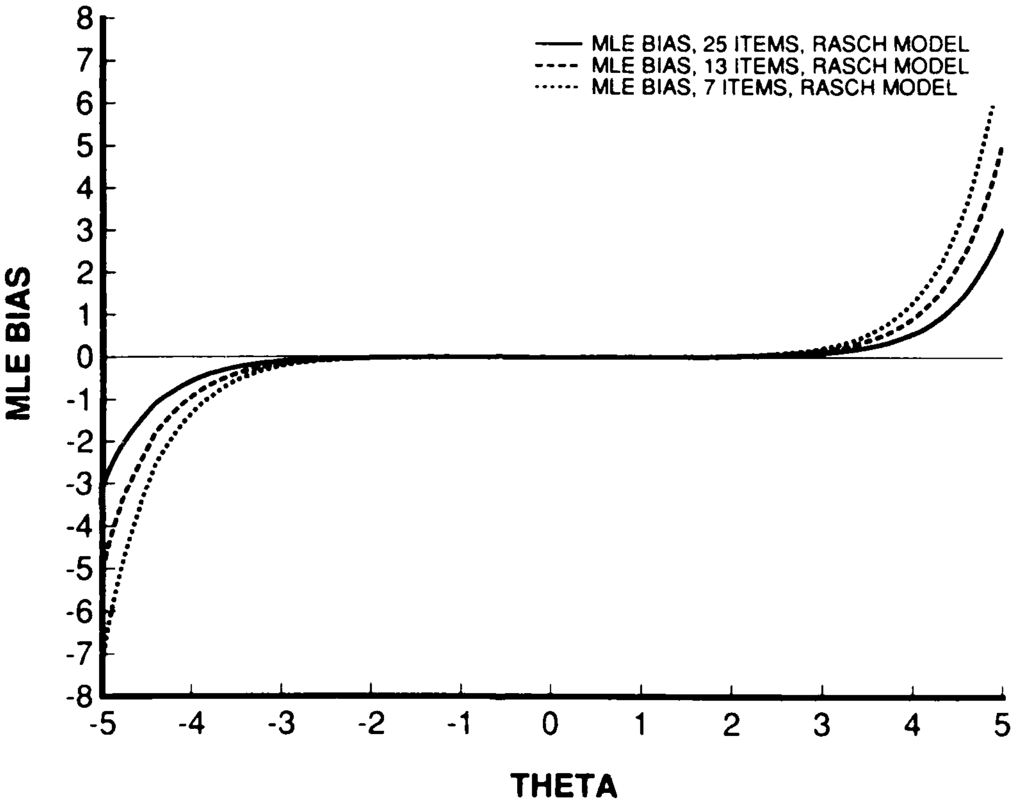
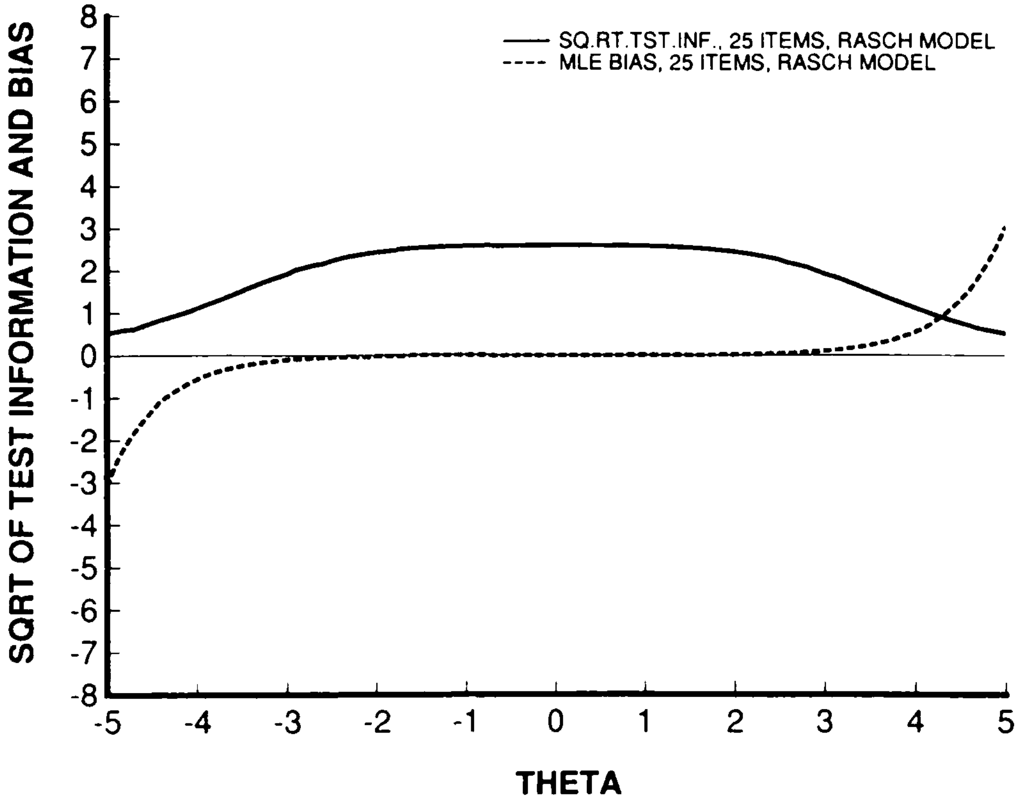


FIGURE 3-9

MLE Bias Function of Each of Three Hypothetical Tests of 25 (Solid Line), 13 (Dashed Line) and 7 (Dotted Line) Items, Respectively, Following Rasch Model with $D = 1.7$ and Equally Spaced Difficulty Parameters Ranging from -3.0 to $+3.0$.


SQ.RT.TST.INF., 25 ITEMS, RASCH MODEL
MLE BIAS, 25 ITEMS, RASCH MODEL
SQRT OF TEST INFORMATION AND BIAS
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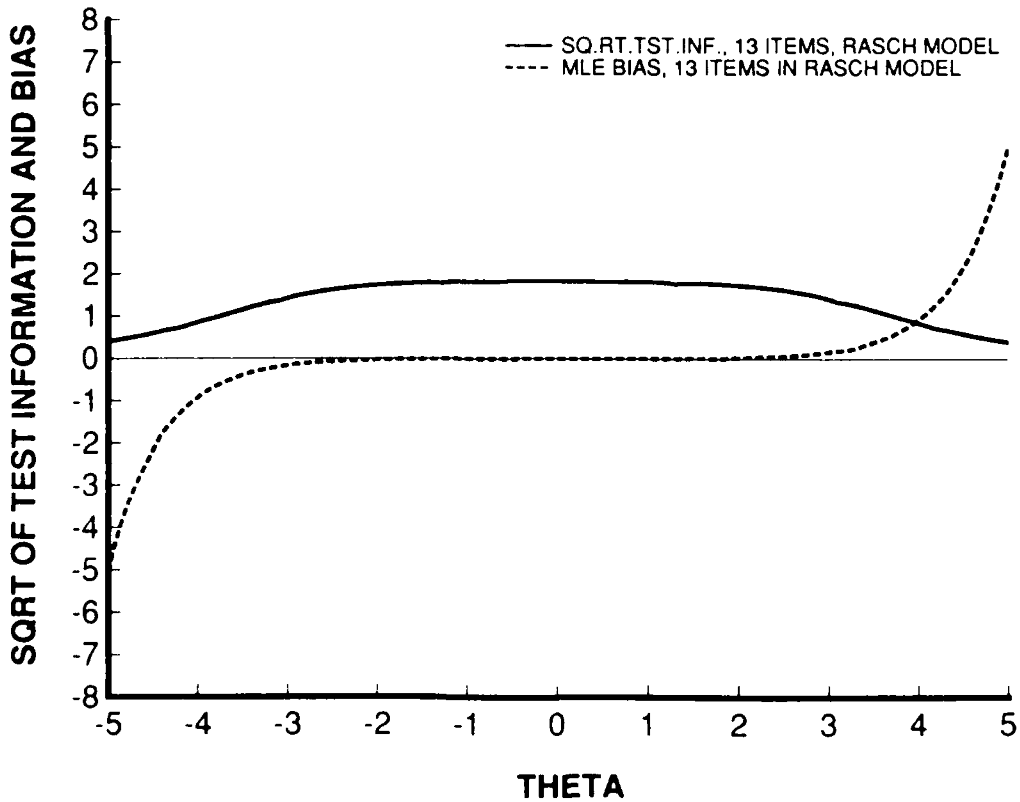
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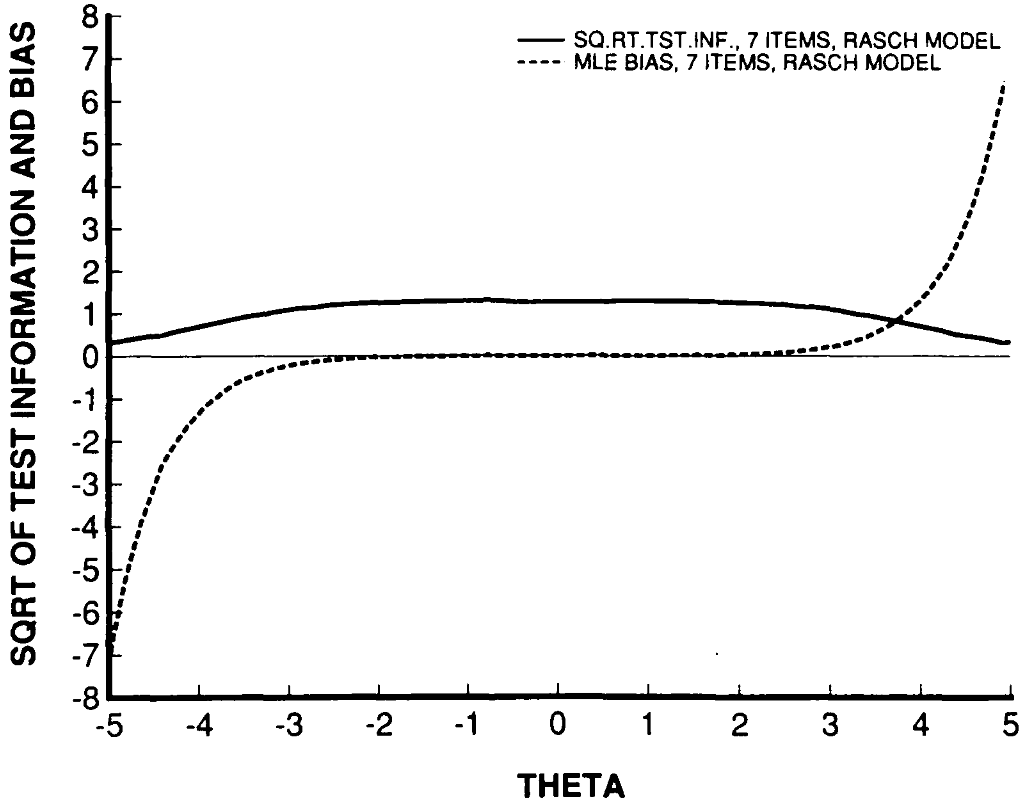


FIGURE 3-10

MLE Bias Function (Dashed Line) and the Square Root of Test Information (Solid Line) of Each of Three Hypothetical Tests of 25 (Solid Line), 13 (Dashed Line) and 7 (Dotted Line) Items, Respectively, Following Rasch Model with $D = 1.7$ and Equally Spaced Difficulty Parameters Ranging from -3.0 to $+3.0$.

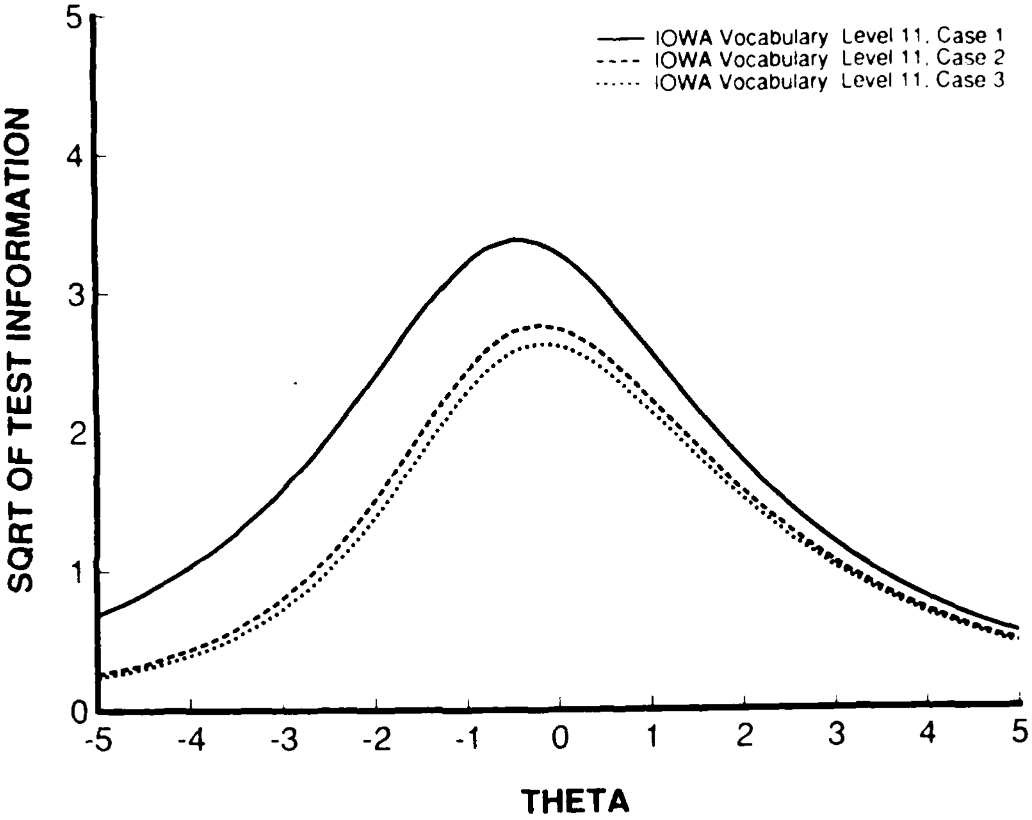


FIGURE 3-11

Square Roots of Test Information of the Iowa Level 11 Vocabulary Subtest (Solid Line) of 43 Items Following the Logistic Model, and of Two Hypothetical Tests of the Same Number of Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as the Iowa Subtest and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.

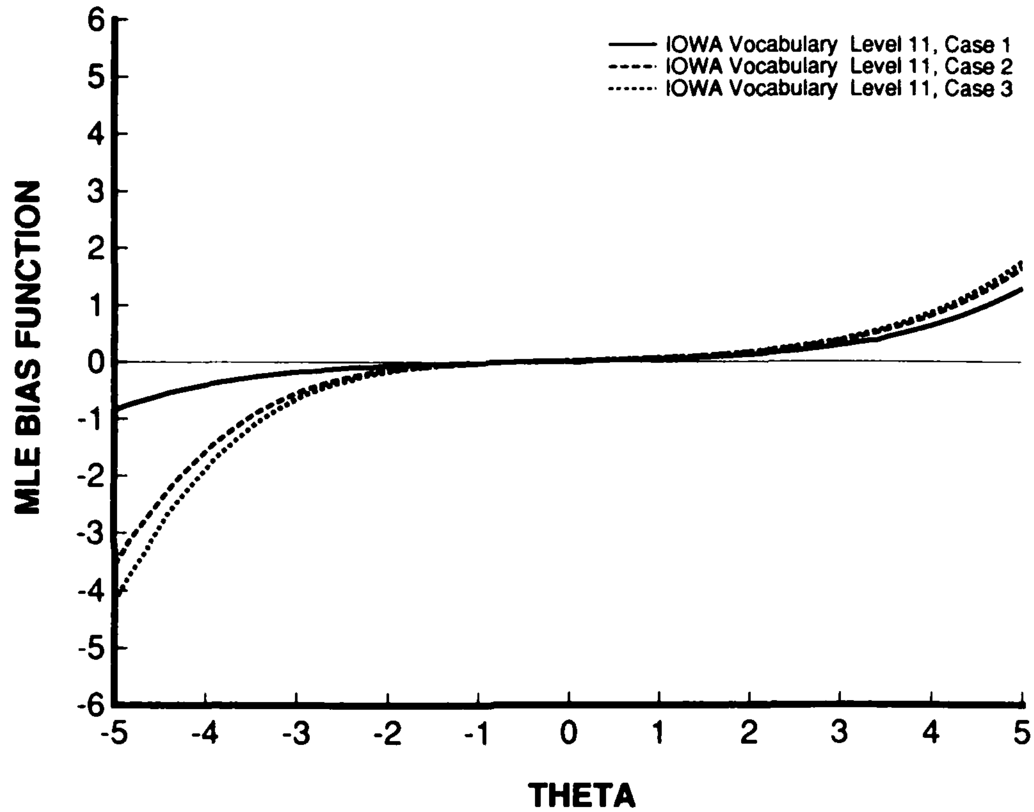
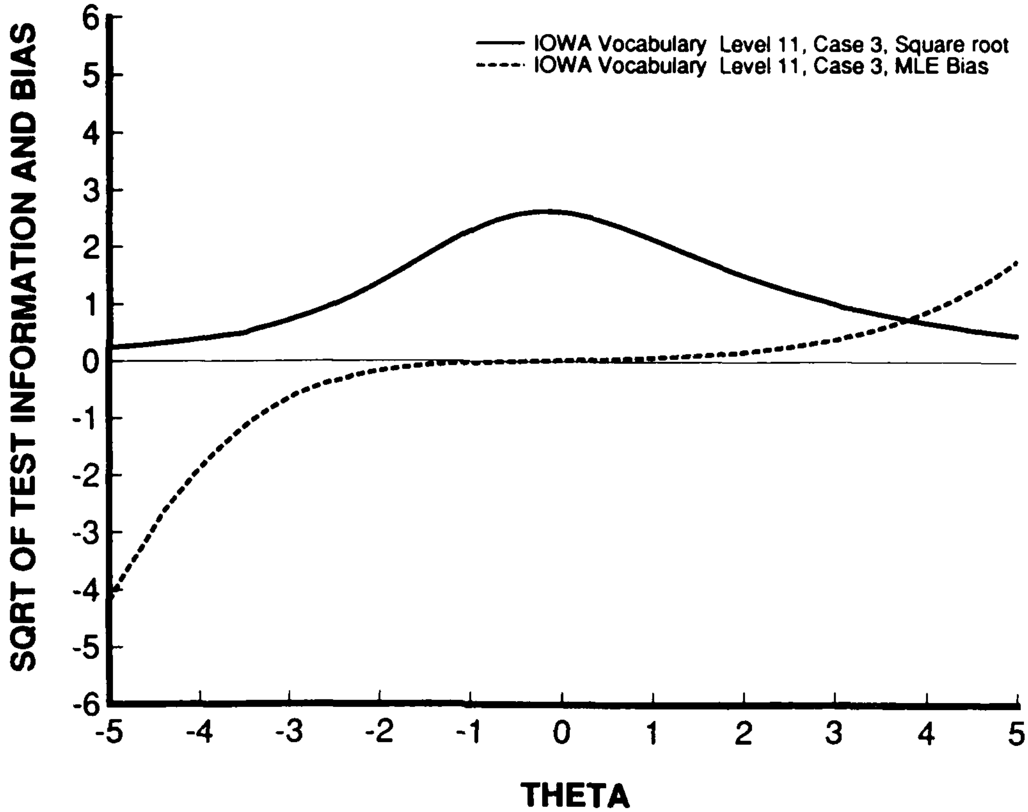


FIGURE 3-12

MLE Bias Functions of the Iowa Level 11 Vocabulary Subtest of 43 Items Following the Logistic Model, and of Two Hypothetical Tests of the Same Number of Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as the Iowa Subtest and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.



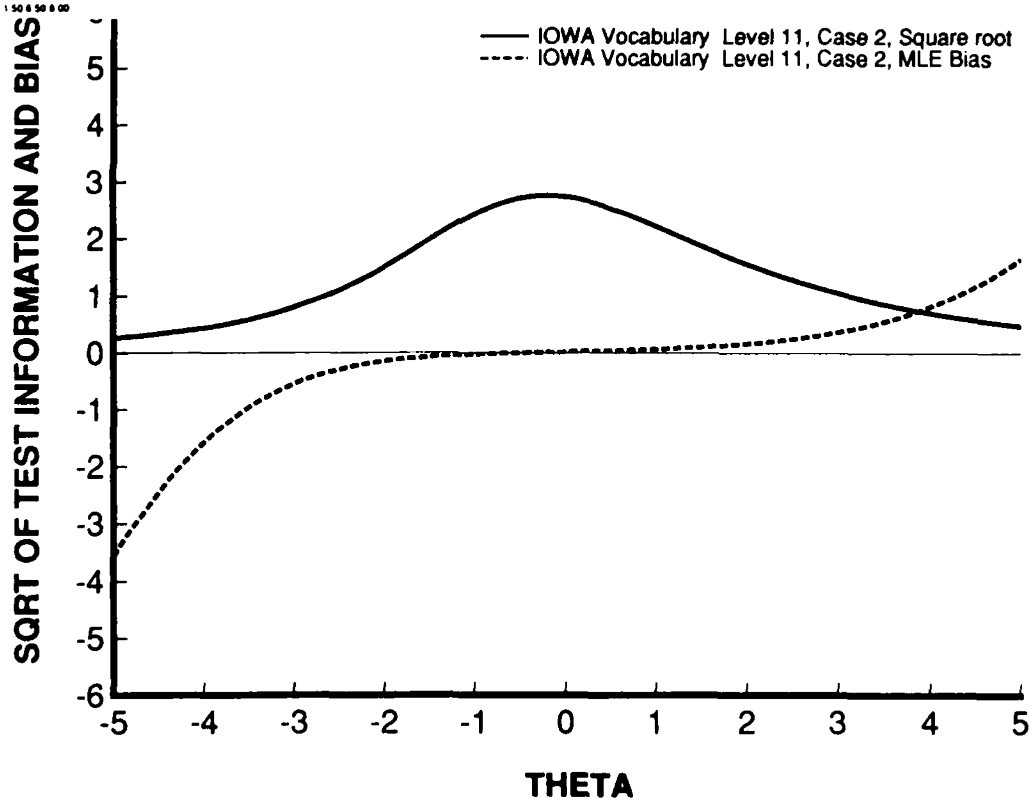


FIGURE 3-13

MLE Bias Function (Dashed Line) and the Square Root of Test Information (Solid Line) of Each of the Two Hypothetical Tests of 43 Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as the Iowa Subtest and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.

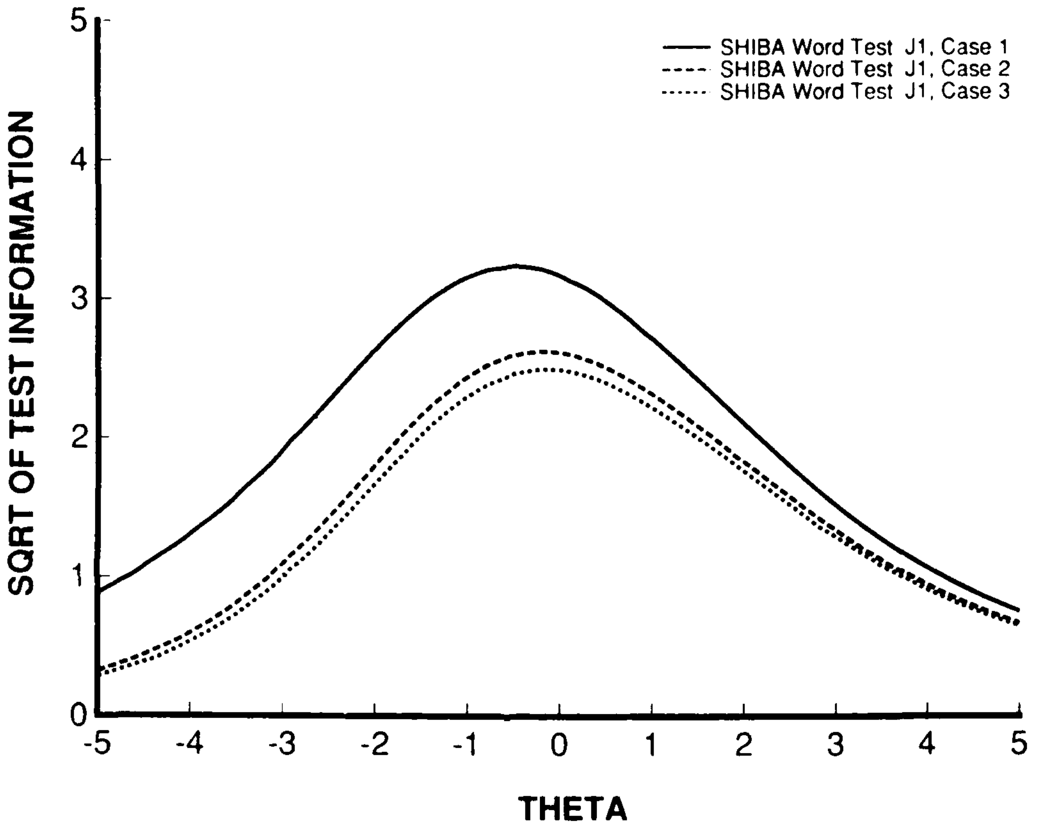


FIGURE 3-14

Square Roots of Test Information of Shiba's Word/Phrase Comprehension Test J1 (Solid Line) of 55 Items Following the Logistic Model, and of Two Hypothetical Tests of the Same Number of Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as Test J1 and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.

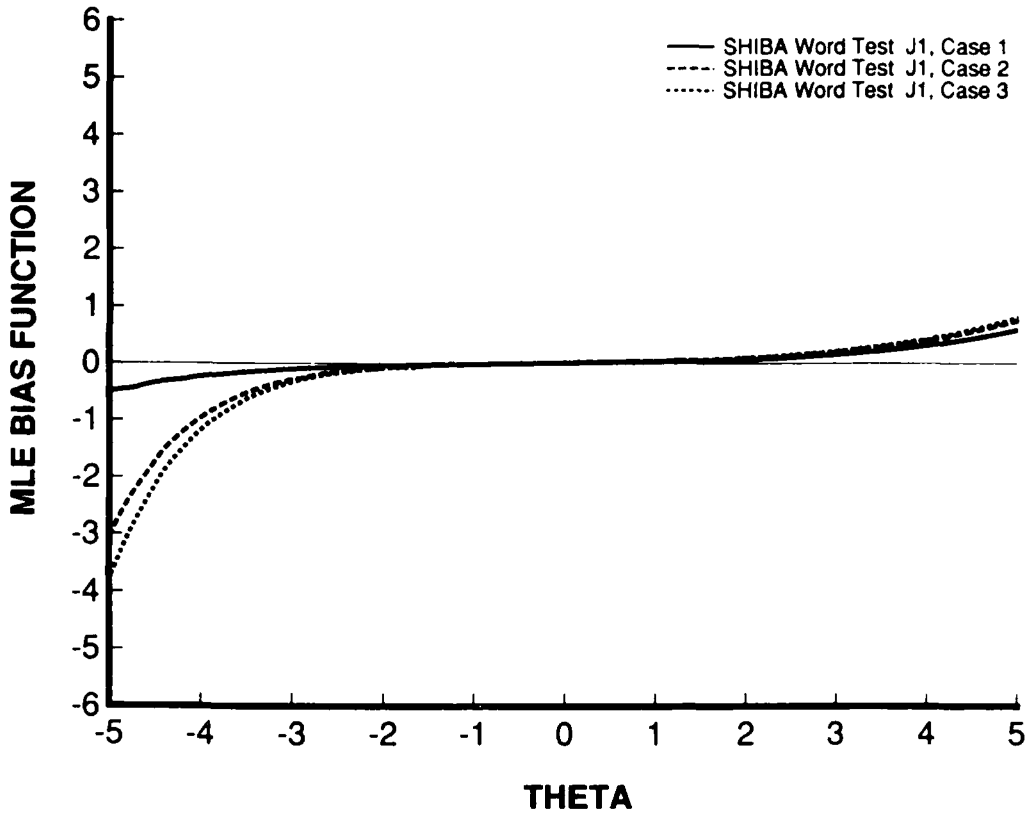


FIGURE 3-15

MLE Bias Functions of Shiba's Word/Phrase Comprehension Test J1 of 55 Items Following the Logistic Model, and of Two Hypothetical Tests of the Same Number of Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as Test J1 and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.

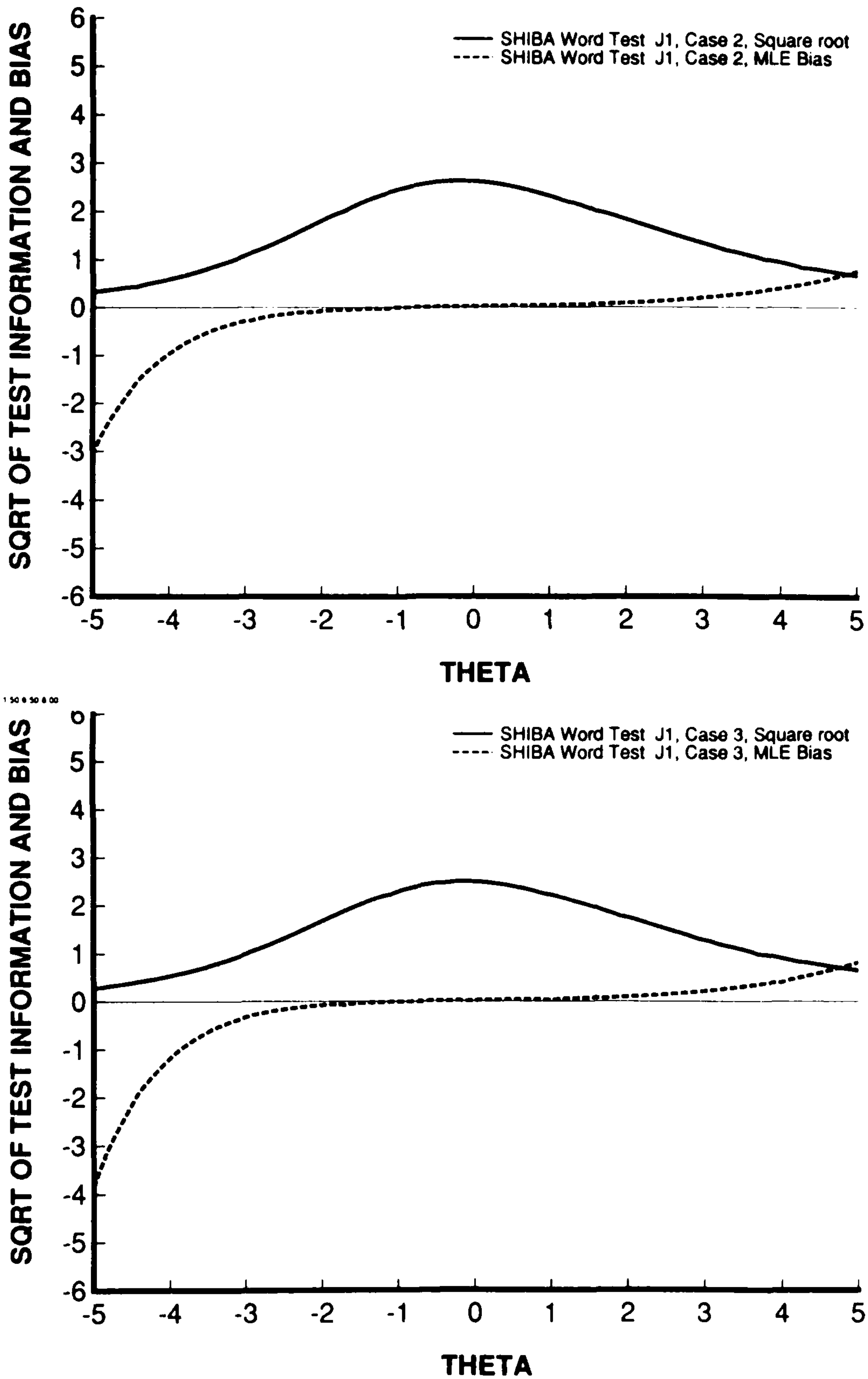


FIGURE 3-16

MLE Bias Function (Dashed Line) and the Square Root of Test Information (Solid Line) of Each of the Two Hypothetical Tests of 55 Items Each Following the Three-Parameter Logistic Model, which Share the Same Set of Item Discrimination and Difficulty Parameters as Test J1 and with the Guessing Parameters 0.20 (Dashed Line) and 0.25 (Dotted Line), Respectively.

results of the Iowa Level 11 Vocabulary Subtest, and similar conclusions can be reached.

4 Graded Response Level

On the graded response level, the bias function is directly given by (2.43) by replacing the general discrete response k_g to item g by the graded item score x_g $(= 0, 1, \ldots, m_g)$.

In the homogenous case of the graded response level (Samejima, 1972), the general formula for the operating characteristic of the item score x_g is given by

(4.1) $$P_{x_g}(\theta) = P^*_{x_g}(\theta) - P^*_{(x_g+1)}(\theta) \quad ,$$

where

(4.2) $$P^*_{x_g}(\theta) = \int_{-\infty}^{a_g(\theta - b_{x_g})} \psi_g(t)\,dt \quad ,$$

(4.3) $$-\infty = b_0 < b_1 < b_2 < \ldots < b_{m_g} < b_{m_g+1} = \infty \quad ,$$

and $\psi_g(\theta)$ is some specified density function. When we replace the right hand side of (4.2) by that of (3.1) with b_g replaced by b_{x_g} , we have the operating characteristic of x_g in the normal ogive model on the graded response level; when we do a similar thing by using the right hand side of (1.5), we obtain the operating characteristic of x_g in the logistic model on the graded response level.

Since, in general, the graded item is more informative than the dichotomous item, we can expect smaller amounts of bias on the graded response level than on the dichotomous response level. Although the relationship between the configuration of the difficulty parameters of the n items and the amount of bias is more complicated on the graded response level, it will be easier in practice to develop a set of items which provides us with negligibly small amounts of bias for a wide range of θ .

We shall see some examples here. In the past years, the author has been engaged in developing nonparametric approaches and methods of estimating the operating characteristics, or the conditional probabilities, given ability θ , assigned to separate discrete item responses. In other words, these approaches and methods are based upon no assumptions concerning the mathematical forms of those operating characteristics. In so doing, the asymptotic normal property of the maximum likelihood estimate (MLE), i.e., the fact that, as the number of items increases, the conditional distribution of MLE, given θ , approaches normality with θ and the inverse of the square root of the test information function as the two parameters, is fully utilized. A set of simulated data has been used for testing these approaches and methods, in which 35 graded test items following the normal ogive model with three item score categories each are hypothesized as the Old Test (cf. Samejima, 1977, 1981). Table 4-1 presents the item discrimination parameter a_g and the two item response difficulty parameters, i.e., b_{x_g} for $x_g = 1, 2$, for each of the 35 hypothesized items. The square root of the test information function of this Old Test is shown as the solid curve in Figure 4-1. The bias function, which was computed through (2.43), is shown in Figure 4-2 as the solid curve. We can see in this figure that for the interval of θ covering $(-4, 4)$ the bias of the maximum likelihood estimate is practically zero, i.e., the MLE of ability is practically unbiased for this range of θ . Thus one of the necessary conditions to justify the use of the asymptotic normality as the approximation for the conditional distribution of MLE, given θ , is satisfied.

We notice in Figure 4-1 that for the range of θ , $(-3, 3)$, the square root of the test information function of this Old Test assumes approximately a constant value of 4.65 , and we have already seen that for the wider range of θ the bias function assumes, practically, zero. It is interesting to note that

TABLE 4-1

Item Discrimination Parameter a_g and Two Item Difficulty Parameters b_{x_g} , $x_g = 1, 2$, for Each of the Thirty-Five Graded Test Items of the Old Test.

Item g	a_g	b_1	b_2
1	1.8	-4.75	-3.75
2	1.9	-4.50	-3.50
3	2.0	-4.25	-3.25
4	1.5	-4.00	-3.00
5	1.6	-3.75	-2.75
6	1.4	-3.50	-2.50
7	1.9	-3.00	-2.00
8	1.8	-3.00	-2.00
9	1.6	-2.75	-1.75
10	2.0	-2.50	-1.50
11	1.5	-2.25	-1.25
12	1.7	-2.00	-1.00
13	1.5	-1.75	-0.75
14	1.4	-1.50	-0.50
15	2.0	-1.25	-0.25
16	1.6	-1.00	0.00
17	1.8	-0.75	0.25
18	1.7	-0.50	0.50

Item g	a_g	b_1	b_2
19	1.9	-0.25	0.75
20	1.7	0.00	1.00
21	1.5	0.25	1.25
22	1.8	0.50	1.50
23	1.4	0.75	1.75
24	1.9	1.00	2.00
25	2.0	1.25	2.25
26	1.6	1.50	2.50
27	1.7	1.75	2.75
28	1.4	2.00	3.00
29	1.9	2.25	3.25
30	1.6	2.50	3.50
31	1.5	2.75	3.75
32	1.7	3.00	4.00
33	1.8	3.25	4.25
34	2.0	3.50	4.50
35	1.4	3.75	4.75

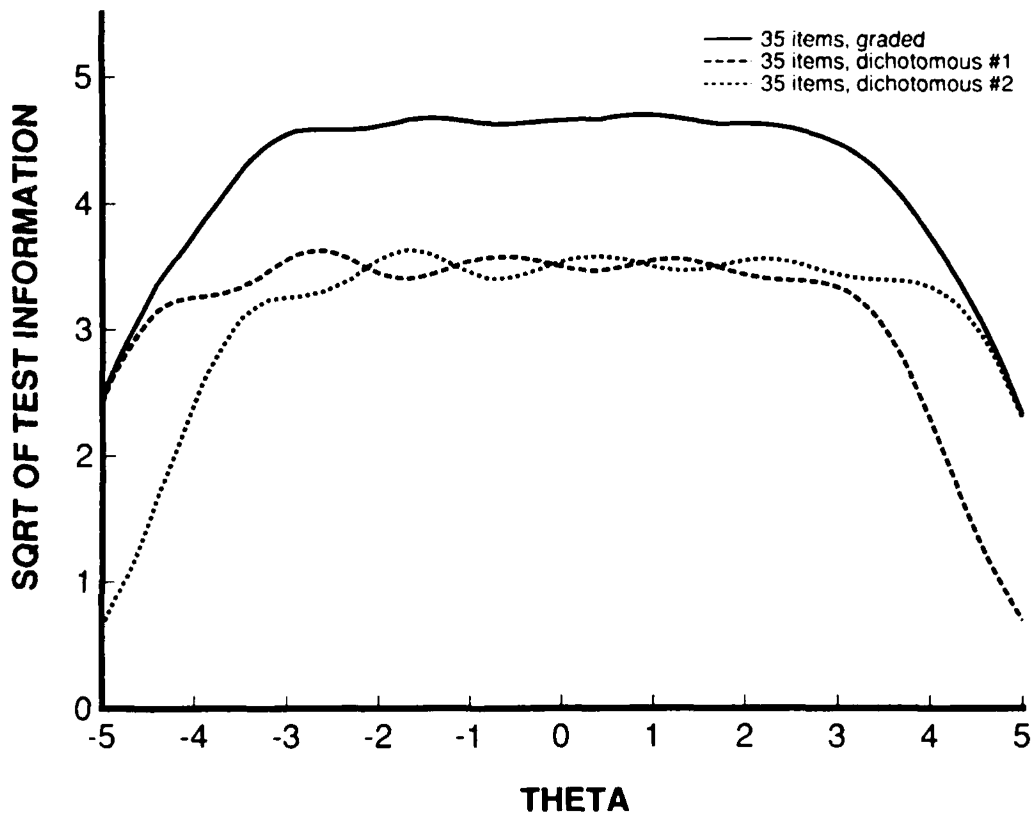


FIGURE 4-1

Square Roots of the Test Information Functions for the Old Test of 35 Graded Items (Solid Line), for the Two Sets of 35 Redichotomized Items Using the First Set (Dashed Line) and Second Set (Dotted Line) of the Difficulty Parameters of the Old Test, Respectively, Following the Normal Ogive Method.

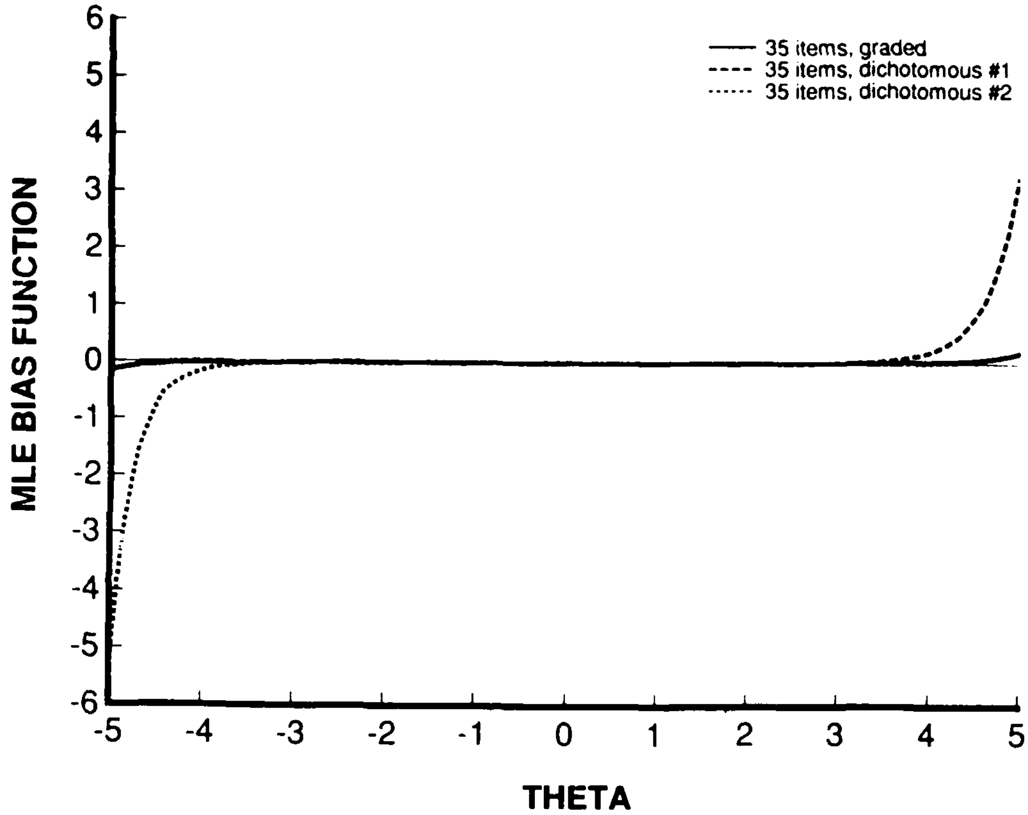
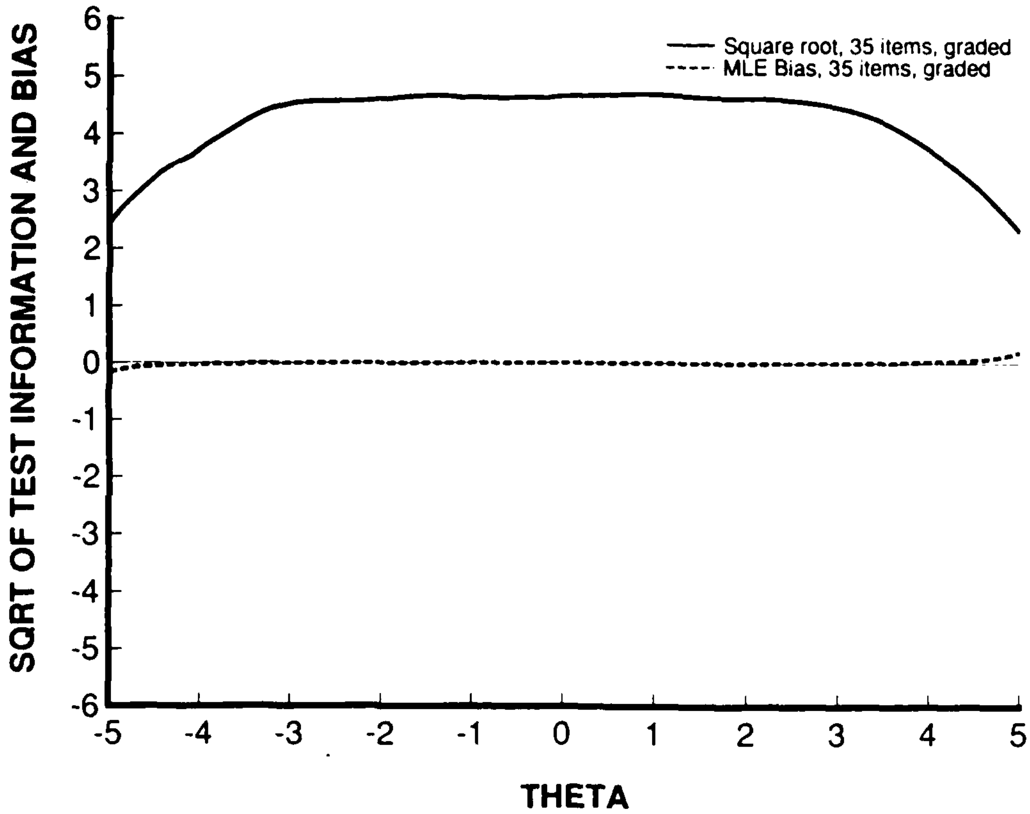
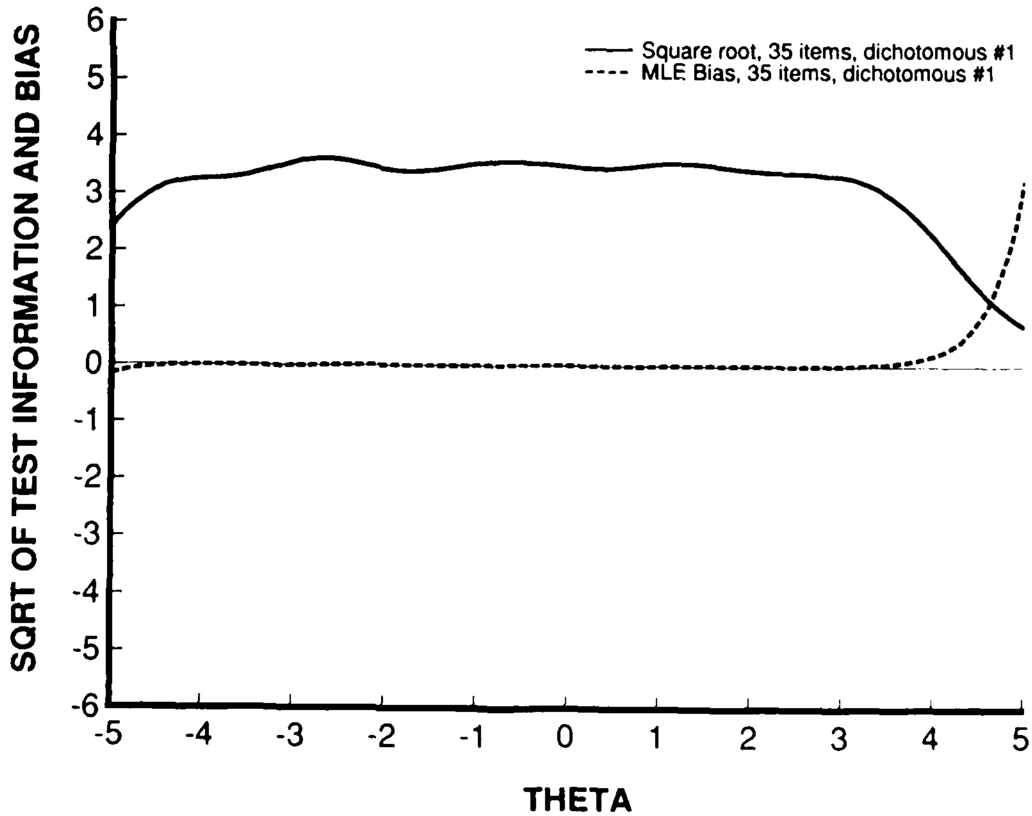


FIGURE 4-2

Bias Functions of the Old Test of 35 Graded Items (Solid Line) and of the Two Sets of 35 Redichotomized Items Using the First Set (Dashed Line) and the Second Set (Dotted Line) of the Difficulty Parameters of the Old Test, Respectively, Following the Normal Ogive Model.


Square root, 35 items, graded
MLE Bias, 35 items, graded
SQRT OF TEST INFORMATION AND BIAS
THETA


Square root, 35 items, dichotomous #1
MLE Bias, 35 items, dichotomous #1
SQRT OF TEST INFORMATION AND BIAS
THETA

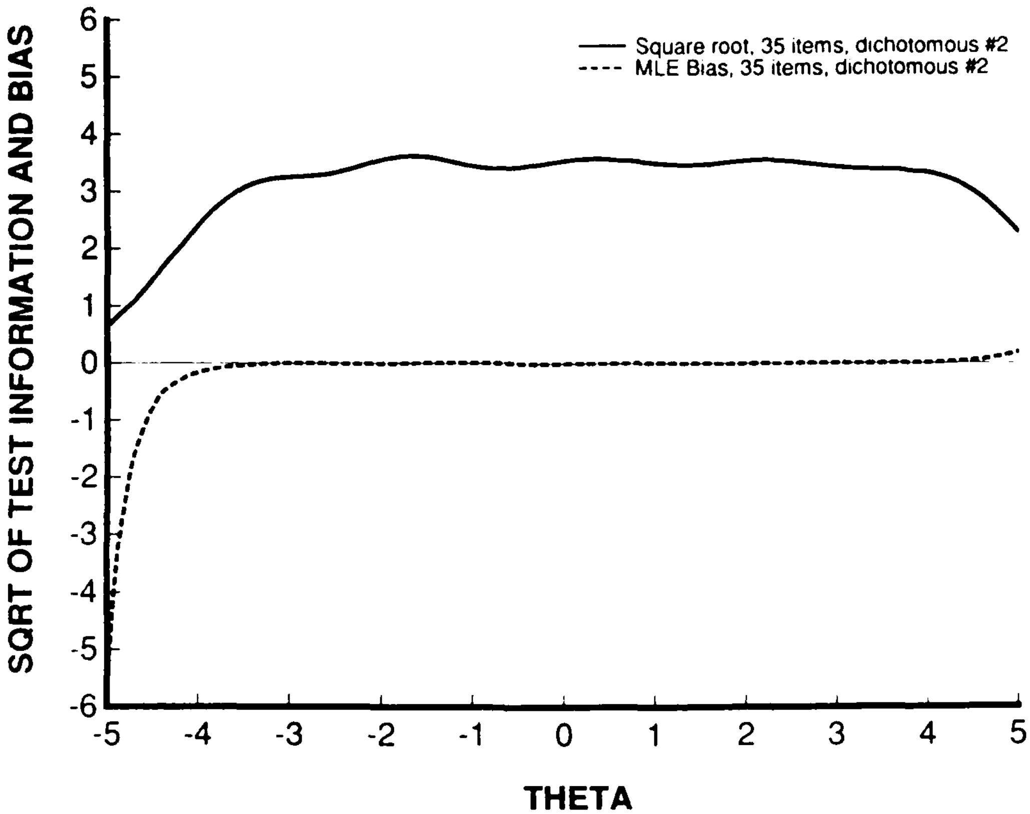


FIGURE 4-3

Comparison of the Square Root of the Test Information Function (Solid Line) and the Bias Function (Dashed Line) of Each of the Three Tests, i.e., the Old Test (First Graph) and the Two Sets of Redichotomized Items Using the First (Second Graph) and the Second (Third Graph) Sets of Difficulty Parameters, Respectively.

the bias starts showing up both positively and negatively when the square root of test infomation drops lower than a critical value, which is approximately 3.2 , or the test information function drops lower than approximately 10 . In order to pursue this relationship, two more sets of these two functions are also shown by dashed and dotted curves in Figures 4-1 and 4-2. These two sets were created by redichotomizing the graded items of the Old Test, using the first and second sets of the difficulty parameters in Table 4-1, respectively. We can see that for the wide range of θ the square root of test information is substantially less than that of the original Old Test, which is the natural consequence of redichotomizing the items. It is noticed that for each of these two, the square root of the test information function is barely greater than 3.2 for a wide range of θ , and the bias is still practically nil. Again the bias appears both positively and negatively when the square root of the test information function drops lower than approximately 3.2 . In order to make this observation easier, both the square root of the test information function and the bias function are plotted together in Figure 4-3 for each of the three hypothetical tests, by solid and dashed lines, respectively. If we tolerate biases of up to ± 0.1 , as we did earlier, then the critical value of the square root of test information will approximately be 2.75 , or that of the test information function approximately 7.5 . When the square root of test information drops to less than 2.0 , the bias turns out to be substantially large.

It is interesting to note that, in these examples, the amount of information required to make the bias negligibly small is larger than those observed in the previous examples, i.e., those of Iowa Level 11 Vocabulary Subtest and Shiba's Test J1. This has something to do with the fact that in the Old Test there are only 35 test items with the average discrimination parameter as high as 1.70 , while there are as many as 43 and 55 items in the Iowa Level 11 Vocabulary Subtest and Shiba's J1 Test with the average values of discrimination parameters 0.601 and 0.538 , respectively. We shall investigate the effect of discrimination parameters on the amount of bias from a somewhat different angle in the following section.

5 Effect of the Discrimination Parameter

In the previous sections, we have seen from several examples that, if the amount of test information is substantially large, the amount of bias is negligibly small, and it looks as if there were a critical value of the square root of test information to realize this approximate unbiasedness of the maximum likelihood estimate. This critical value differs from test to test, however, as we have observed in the examples of the Old Test, the Iowa Level 11 Vocabulary Subtest and Shiba's Test J1. On the dichotomous response level, the effect of the configuration of the difficulty parameters in a test can be seen fairly straightforwardly from the formulae of the bias function for different mathematical models, as we have observed earlier. In this section, we shall pursue the effect of the discrimination parameters on the bias function. In so doing, we choose tests of equivalent items, i.e., each of which consists of test items whose item characteristic functions are identical. We have seen earlier that in such a case, a substantial amount of bias starts showing up as θ departs from the common value of difficulty parameters.

In order to simplify the notation, in this section, we use P , Q , I_g , ψ and ϕ to indicate $P_g(\theta)$, $Q_g(\theta)$, $I_g(\theta)$, $\Psi_g(\theta)$ and

$$\phi = \phi_g\{a(\theta - b)\} = (2\pi)^{-1/2} e^{-a^2(\theta-b)^2/2} , \tag{5.1}$$

which are common for all the n items, where $a = a_1 = \ldots = a_n$ and $b = b_1 = \ldots = b_n$. In the normal ogive model, (3.4) can be simplified for the set of n equivalent items so that we obtain

$$B(\theta) = (1/2)PQ(\theta - b)n^{-1}\phi^{-2} , \tag{5.2}$$

because of (1.6), (1.7) and (3.2). We have for the partial derivative of $B(\theta)$ with respect to a

$$\frac{\partial}{\partial a}B(\theta) = (2n)^{-1}(\theta - b)\phi^{-4}[\phi^2\frac{\partial}{\partial a}(PQ) - 2\phi PQ\frac{\partial}{\partial a}\phi] \ . \tag{5.3}$$

By virtue of the fact that

$$\frac{\partial}{\partial a}(PQ) = \phi(\theta - b)(Q - P) \tag{5.4}$$

and

$$\frac{\partial}{\partial a}\phi = \phi[-a(\theta - b)^2] \ , \tag{5.5}$$

we can write for the last factor of the right hand side of (5.3)

$$\phi^2\frac{\partial}{\partial a}(PQ) - 2\phi PQ\frac{\partial}{\partial a}\phi = 2\phi^2(\theta - b)PQ[a(\theta - b) - \frac{1}{2}\{-\frac{\phi}{P} + \frac{\phi}{Q}\}] \ . \tag{5.6}$$

We notice that the second term on the parenthesis on the right hand side of (5.6) equals $\frac{1}{2}[E[u \mid u < a(\theta - b)] + E[u \mid u \geq a(\theta - b)]]$, since we have

$$\begin{aligned} E[u \mid u < a(\theta - b)] &= \int_{-\infty}^{a(\theta-b)} u\phi(u)\, du/P \\ &= -\phi/P \ , \\ &= [-\phi(u) \mid_{-\infty}^{a(\theta-b)}]/P \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} E[u \mid u \geq a(\theta - b)] &= \int_{a(\theta-b)}^{\infty} u\phi(u)\, du/Q \\ &= [-\phi(u) \mid_{a(\theta-b)}^{\infty}]/Q \\ &= \phi/Q \ . \end{aligned} \tag{5.8}$$

It is obvious that this average of the two expectations of u equals zero when $\theta = b$ and assumes negative and positive values when $\theta < b$ and $\theta > b$, respectively. In addition, we obtain

$$\frac{1}{2}\{-\frac{\phi}{P} + \frac{\phi}{Q}\} \begin{cases} > a(\theta - b) & \theta < b \\ = a(\theta - b) & \theta = b \\ < a(\theta - b) & \theta > b \ . \end{cases} \tag{5.9}$$

To prove this, since we can write for the item information function in the normal ogive model

$$I_g = a^2\phi^2(PQ)^{-1} \ , \tag{5.10}$$

its first derivative I'_g with respect to θ is given by

$$(5.11) \qquad I'_g = 2aI_g[\frac{1}{2}\{-\frac{\phi}{P}+\frac{\phi}{Q}\} - a(\theta-b)] \ .$$

Setting (5.11) equal to zero, we obtain $\theta = b$. From (5.11),we can see that the second derivative I''_g of the item information function with respect to θ assumes $4a^2\pi^{-2}(2-\pi)$ at $\theta = b$, which is negative. Thus $\theta = b$ is the point of θ at which I_g is maximal, and I'_g assumes positive values for $\theta < b$ and negative values for $\theta > b$. Equation (5.9) is the direct consequence of this fact.

Figure 5-1 presents the square root of the test information function for each of the five examples following the normal ogive model by a solid line and dashed lines of various lengths. In these five examples, the numbers of equivalent items are uniformly 30, and the common values of the difficulty parameter are all 0.0 . The common values of the discrimination parameter differ for different tests, however, i.e., they assume 0.4 , 0.7 , 1.0 , 1.5 and 2.0 , respectively. The five bias functions for these five hypothetical tests are shown in Figure 5-2, using the same set of solid and dashed lines.

We can see in these figures how rapidly the amount of bias increases when the common discrimination parameter is large, in both negative and positive directions as θ departs from zero, at which the square root of test information is maximal, especially when $a_g = 2.0$. It is also noted that, taking the criterion of ± 0.1 again, in order to keep the practical unbiasedness the square root of test information must be as large as 4.0 when $a_g = 2.0$, while it can be as small as 1.3 when $a_g = 0.4$. For the intermediate values of the discrimination parameter, i.e., for $a_g = 0.7,\ 1.0,\ 1.5$, the corresponding criterion values of the square root of test information are approximately 2.0 , 2.5 and 3.0 , respectively.

In the logistic model, we can rewrite (1.4) for the set of n equivalent items to obtain

$$(5.12) \qquad \begin{aligned} B(\theta) &= DnaI_g\{\Psi - \frac{1}{2}\}[nI_g]^{-2} \\ &= \{\Psi - \frac{1}{2}\}[nDa\Psi(1-\Psi)] \ , \end{aligned}$$

by virtue of (1.3), (1.5), (1.6) and the fact that

$$(5.13) \qquad \Psi'_g(\theta) = Da_g\Psi_g(\theta)[1-\Psi_g(\theta)] \ ,$$

where $\Psi'_g(\theta)$ indicates the first derivative of $\Psi_g(\theta)$ with respect to θ . Since we have

$$(5.14) \qquad \frac{\partial}{\partial a_g}\Psi_g(\theta) = D(\theta - b_g)\Psi_g(\theta)[1-\Psi_g(\theta)] \ ,$$

the numerator of the partial derivative of $B(\theta)$ with respect to a can be written as

$$(5.15) \qquad \begin{aligned} &nD^2a\Psi^2[1-\Psi]^2(\theta-b) - [\Psi - \frac{1}{2}]nD^2a\Psi[1-\Psi][1-2\Psi](\theta-b) \\ &= nD^2a\Psi[1-\Psi][\frac{1}{2} - \Psi\{1-\Psi\}](\theta-b). \end{aligned}$$

Since all the factors on the right hand side of (5.15) are positive except for the last one, we can conclude that the amount of bias equals zero at $\theta = b$ regardless of the value of a , and increases in the positive and negative directions for $\theta > b$ and $\theta < b$, respectively.

Figures 5-3 and 5-4 present the square root of the test information function and the bias function for each of the five hypothetical tests following the logistic model, respectively, which share the same number of items and the parameter values as those five hypothetical tests following the normal ogive model. These results are very similar to those obtained for the normal ogive model, except for the fact that the intervals of practical unbiasedness are a little smaller.

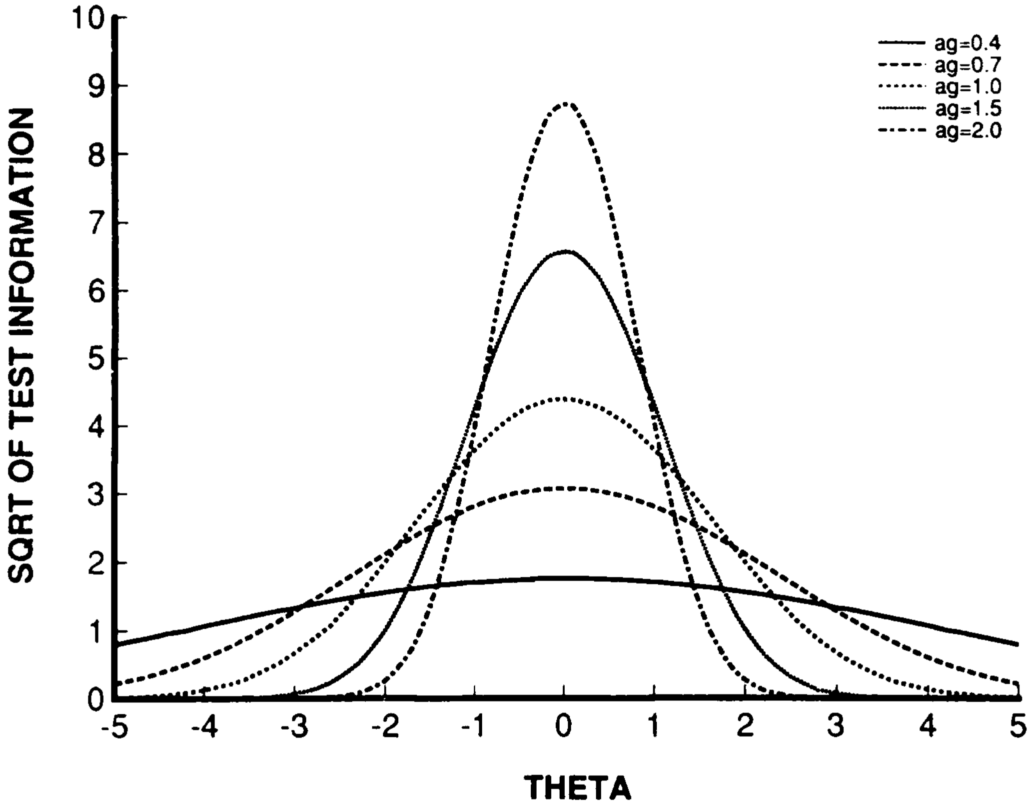


FIGURE 5-1

Square Roots of Test Information for the Five Hypothetical Tests of 30 Equivalent Items Following the Normal Ogive Model. The Common Values of the Difficulty Parameter Are Uniformly 0.0 , and Those of the Discrimination Parameter are 0.4 , 0.7 , 1.0 , 1.5 , and 2.0 , Respectively.

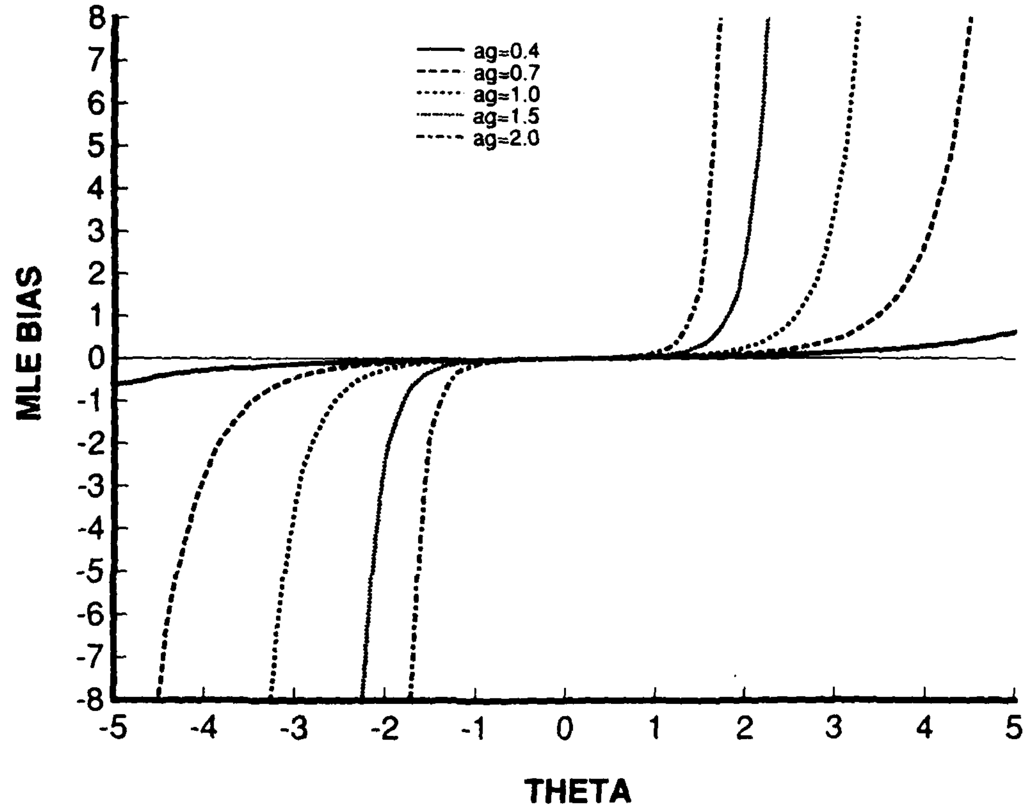


FIGURE 5-2

Bias Functions for the Five Hypothetical Tests of 30 Equivalent Items Following the Normal Ogive Model. The Common Values of the Difficulty Parameter Are Uniformly 0.0 , and Those of the Discrimination Parameter are 0.4 , 0.7 , 1.0 , 1.5 , and 2.0 , Respectively.

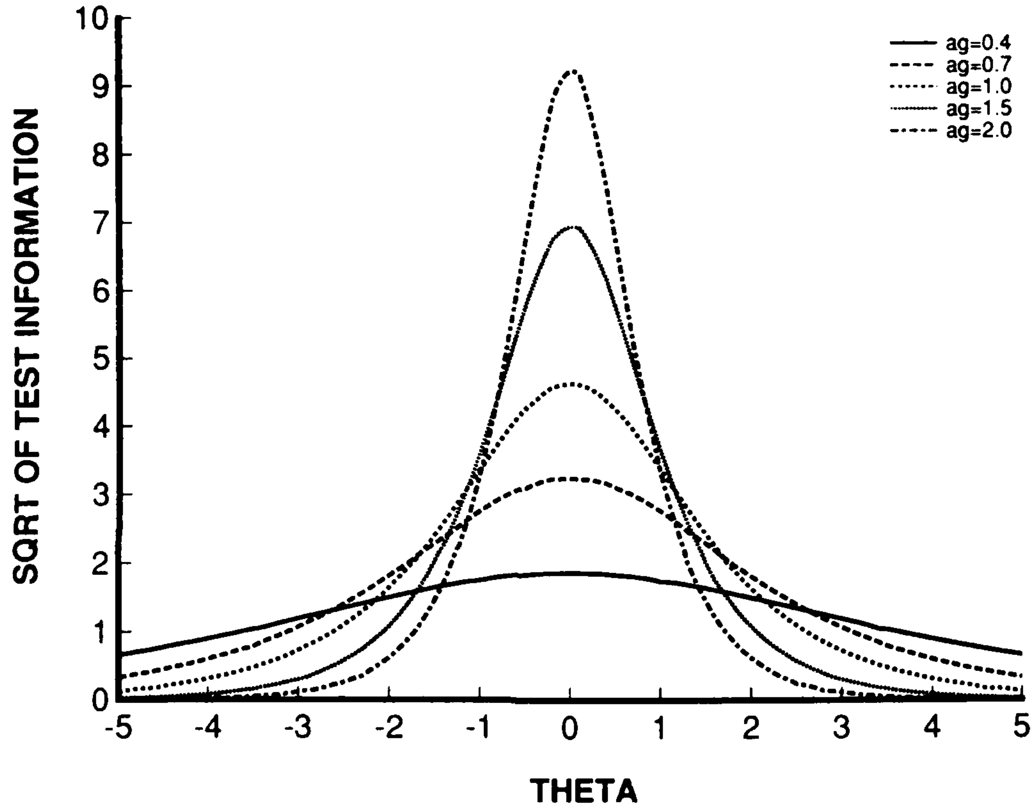


FIGURE 5-3

Square Roots of Test Information for the Five Hypothetical Tests of 30 Equivalent Items Following the Logistic Model. The Common Values of the Difficulty Parameter Are Uniformly 0.0 , and Those of the Discrimination Parameter are 0.4 , 0.7 , 1.0 , 1.5 , and 2.0 , Respectively.

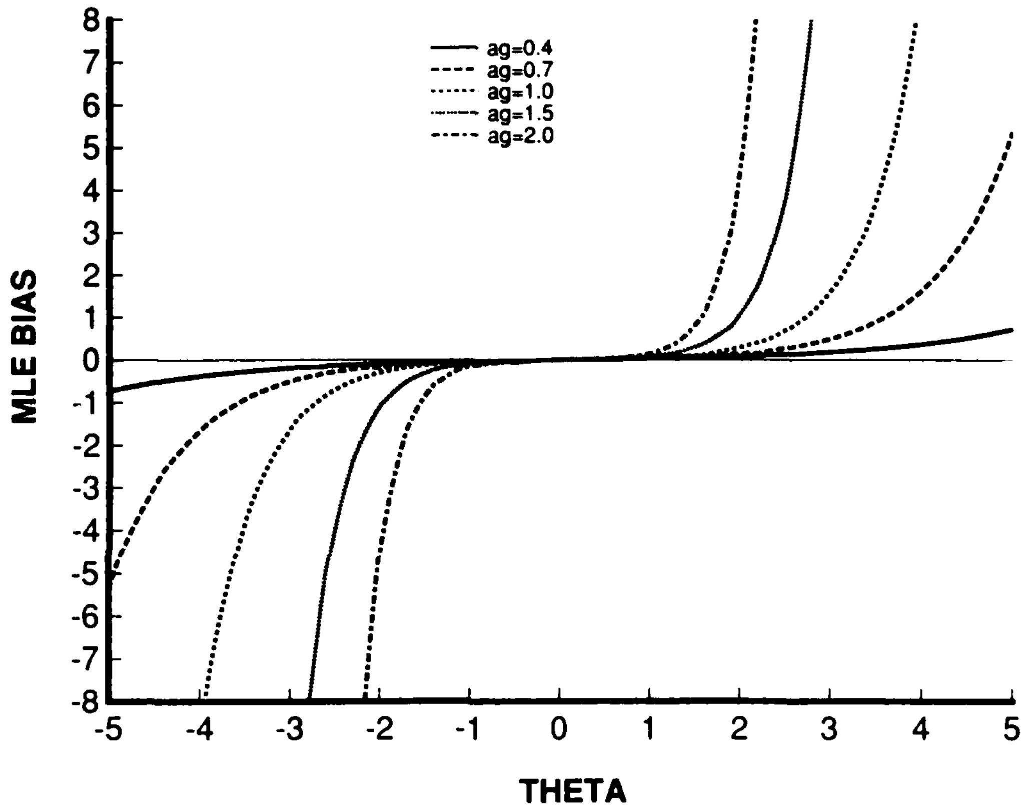


FIGURE 5-4

Bias Functions for the Five Hypothetical Tests of 30 Equivalent Items Following the Logistic Model. The Common Values of the Difficulty Parameter Are Uniformly 0.0 and Those of the Discrimination Parameter are 0.4 , 0.7 , 1.0 , 1.5 and 2.0, Respectively.

6 Effect of the Number of Items

It is obvious from (1.7), (2.43) and (2.46) that the number of items in a test affects the amount of bias through the test information function, in the negative way such that its increase causes the decrease in the amount of bias. We have also observed from our examples that, even if the amounts of test information are the same for two different tests, they may not share the same amount of bias. In this regard, we have seen that the values of the item discrimination parameters affect the amount of bias in the positive direction. It has also been pointed out that the configuration of the difficulty parameters in a test affects the bias function.

In order to demonstrate these effects further, in this section, we shall observe the effect of the number of items using different numbers of equivalent items, each of which follows the constant information model (Samejima, 1979) on the dichotomous response level. The item characteristic function in the constant information model is defined by

$$P_g(\theta) = \sin^2\{\alpha_g(\theta - \beta_g) + (\pi/4)\} \quad . \tag{6.1}$$

where α_g and β_g are the item discrimination and difficulty parameters, respectively. From (6.1) we obtain

$$Q_g(\theta) = \cos^2\{\alpha_g(\theta - \beta_g) + (\pi/4)\} \quad . \tag{6.2}$$

This model provides us with a constant amount of item information, i.e.,

$$I_g(\theta) = 4\alpha_g^2 \quad , \tag{6.3}$$

for the interval of θ such that

$$-\pi(4\alpha_g)^{-1} + \beta_g < \theta < \pi(4\alpha_g)^{-1} + \beta_g \quad . \tag{6.4}$$

Since we have

$$P_g'(\theta) = 2\alpha_g[P_g(\theta)Q_g(\theta)]^{1/2} \tag{6.5}$$

and

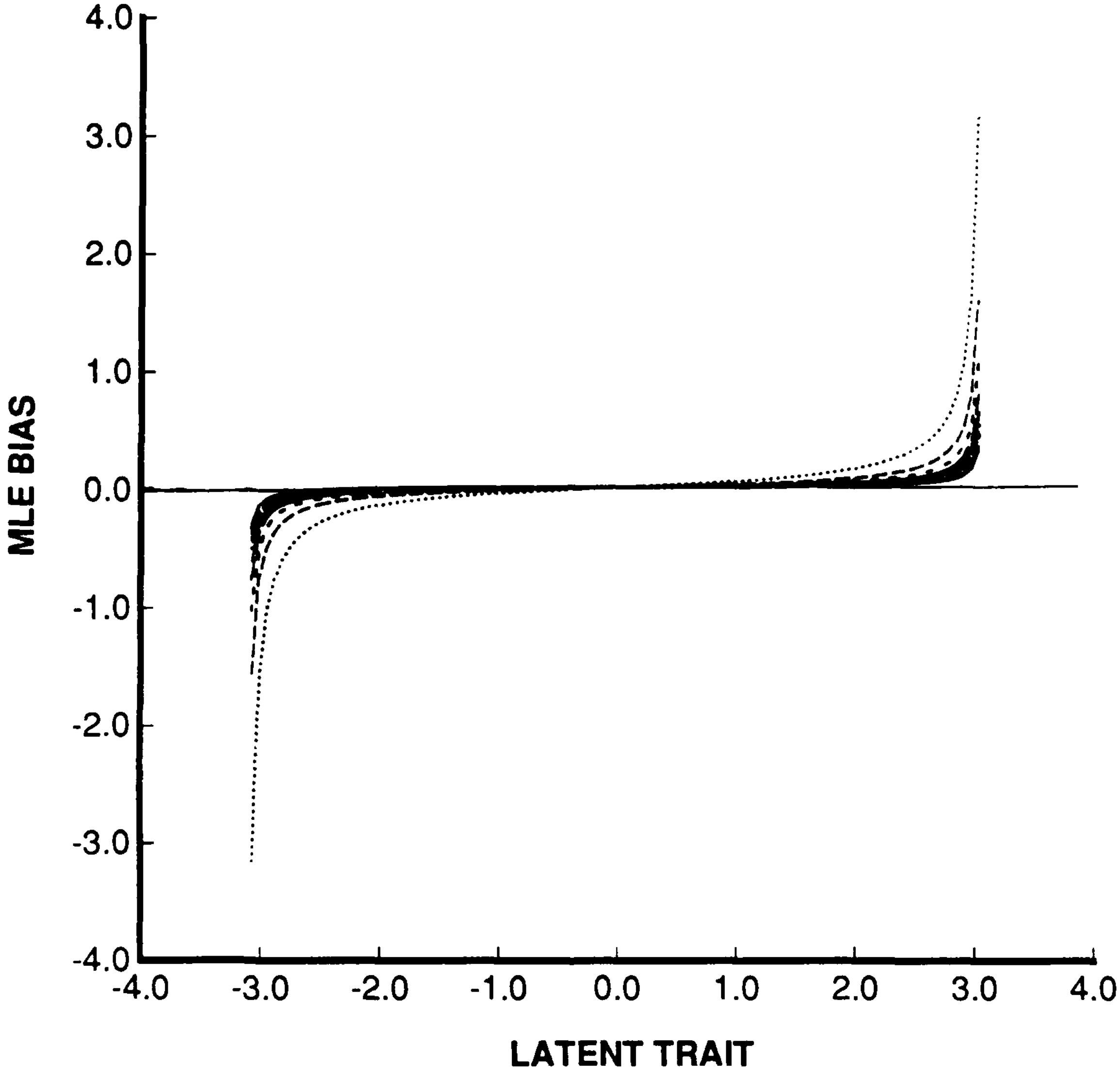
$$P_g''(\theta) = 2\alpha_g^2[Q_g(\theta) - P_g(\theta)] \quad , \tag{6.6}$$

substituting these and (6.3) into (2.46) and rearranging, we can write for the bias function in the constant information model

$$B(\theta) = 2\{I(\theta)\}^{-2}\sum_{g=1}^{n}\alpha_g^3\{P_g(\theta) - Q_g(\theta)\}\{P_g(\theta)Q_g(\theta)\}^{-1/2} \quad . \tag{6.7}$$

For the set on n equivalent items, we can simplify (6.7) into the form

$$B(\theta) = \{8n\alpha_g\}^{-1}\{P_g(\theta) - Q_g(\theta)\}\{P_g(\theta)Q_g(\theta)\}^{-1/2} \quad . \tag{6.8}$$



1.000 0.50 1.00 10.00 6.055

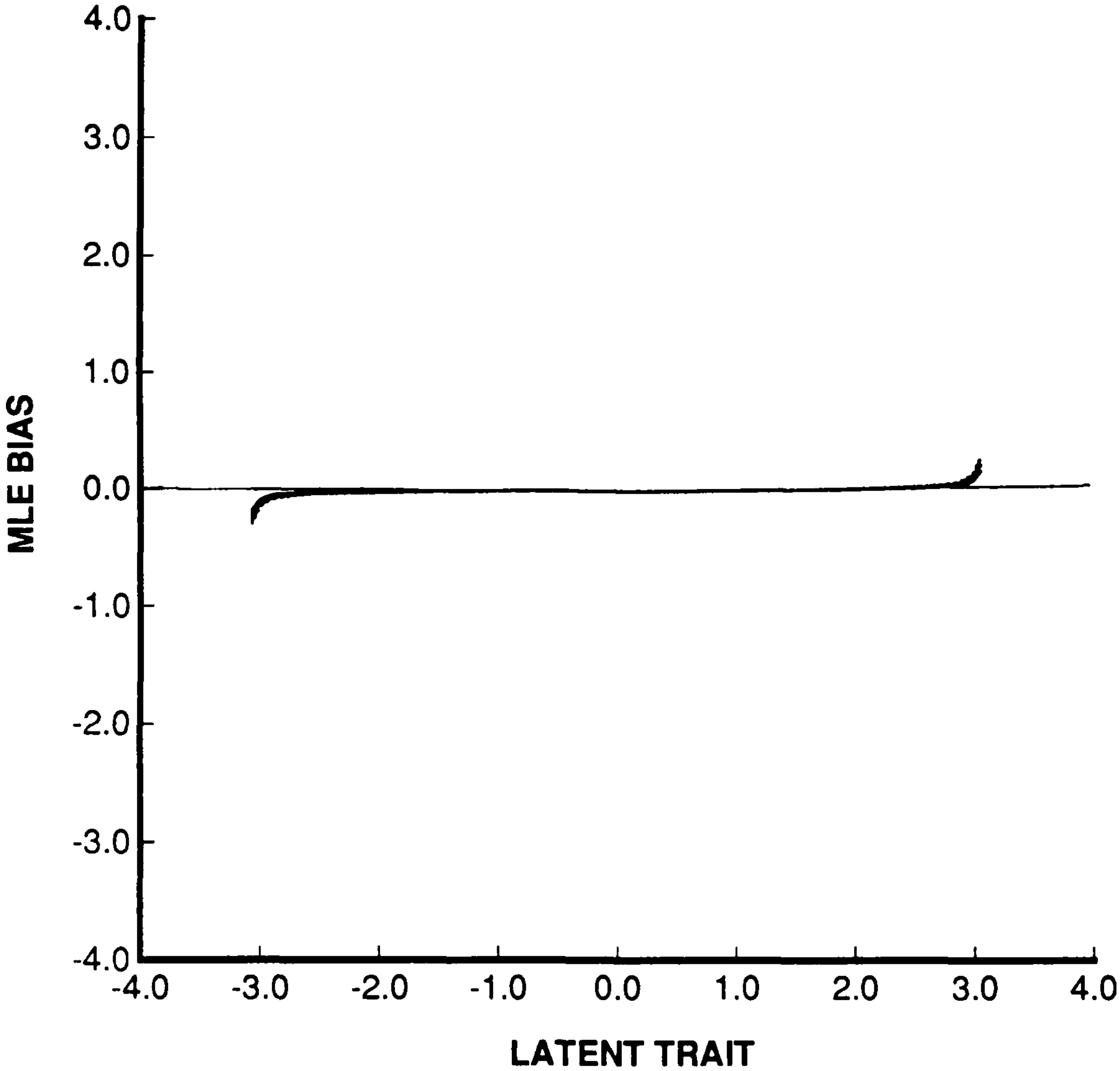


FIGURE 6-1

MLE Bias Function for Each of the Twenty Different Values of n, Which Starts with 10, Increases by 10 Successively, and Ends with 200. The Common Item Parameters for These Equivalent Items Are Given by $\alpha_g = 0.25$ and $\beta_g = 0.00$, and, Therefore, in Each Set of Equivalent Items, the Test Information Function Assumes a Constant Value, $0.25n$, for the Range of θ, $-\pi < \theta < \pi$.

Figure 6-1 presents, in two graphs, this bias function for each of the twenty different values of n, which starts with 10, increases by 10 successively, and ends with 200. The common item parameters for these equivalent items are given by $\alpha_g = 0.25$ and $\beta_g = 0.00$, and, therefore, in each set of equivalent items, the test information function assumes a constant value of $0.25n$ for the range of θ, $-\pi < \theta < \pi$. We can see in this figure that the amount of bias is substantial as θ departs from $\beta_g = 0.00$ when n is relatively small, but it becomes negligibly small for larger values of n, especially when n exceeds 100. These results also illustrate the fact that the amount of test information alone does not control the amount of bias, since they are based upon the constant information model, and the amount of test information is a constant in each set of equivalent items for the range of θ, $-\pi < \theta < \pi$. A usefulness of the constant information model is that we can use it as the benchmark when we deal with a set of equivalent items following such frequently used mathematical models as the normal ogive model, the logistic model, Rasch model, etc. This will be shown with respect to the bias function in the following section.

7 Scale Transformation

It is obvious from (2.43) that the bias function belongs to a particular scale of the latent trait, and, if the scale is transformed, the function changes also. In this section, we shall see how the scale transformation affects the bias function of a test.

7.1 General Case of Discrete Responses

Let τ be a strictly increasing transformation of θ, so that we can write

$$\tau = \tau(\theta) \quad . \tag{7.1}$$

We assume that τ is twice differentiable with respect to θ. Since the operating characteristic, or the conditional probability, of the discrete item response k_g is unchanged for the scale transformation, we can write

$$P^*_{k_g}(\tau) = P_{k_g}(\theta) \quad , \tag{7.2}$$

where $P^*_{k_g}(\tau)$ denotes the operating characteristic of the discrete item response k_g as a function of the transformed latent trait τ. From (7.2) we obtain for the first and second derivatives, $P^{*\prime}_{k_g}(\tau)$ and $P^{*\prime\prime}_{k_g}(\tau)$, of the operating characteristic of k_g with respect to τ

$$P^{*\prime}_{k_g}(\tau) = P'_{k_g}(\theta)\frac{d\theta}{d\tau} \tag{7.3}$$

and

$$P^{*\prime\prime}_{k_g}(\tau) = P''_{k_g}(\theta)[\frac{d\theta}{d\tau}]^2 + P'_{k_g}(\theta)\frac{d^2\theta}{d\tau^2} \quad . \tag{7.4}$$

From (7.3), (7.4) and the definitions of the item response information function and of the item information function, which are given by (1.1) and (1.2), respectively, we can write for the item information function $I^*_g(\tau)$ of item g as a function of the transformed latent trait τ

$$I^*_g(\tau) = I_g(\theta)[\frac{d\theta}{d\tau}]^2 \quad . \tag{7.5}$$

Hence we have for the test information function $I^*(\tau)$ on the transformed scale of the latent trait

(7.6)
$$I^*(\tau) = \sum_{g=1}^{n} I_g^*(\tau) = \sum_{g=1}^{n} I_g[\frac{d\theta}{d\tau}]^2$$
$$= I(\theta)[\frac{d\theta}{d\tau}]^2 \ .$$

We can write from (2.43) for the bias function, $B^*(\tau)$, of a test after the scale transformation

(7.7)
$$B^*(\tau) = -\frac{1}{2}[I^*(\tau)]^{-2} \sum_{g=1}^{n} \sum_{k_g} P_{k_g}^{*\prime}(\tau) P_{k_g}^{*\prime\prime}(\tau) [P_{k_g}^*(\tau)]^{-1} \ .$$

Substituting (7.2), (7.3), (7.4), and (7.6) into (7.7) and rearranging, we obtain

(7.8)
$$B^*(\tau) = B(\theta)[\frac{d\theta}{d\tau}]^{-1} - \frac{1}{2}[I(\theta)]^{-1}[\frac{d\theta}{d\tau}]^{-3}\frac{d^2\theta}{d\tau^2} \ .$$

The bias function of the transformed latent variable can be computed, therefore, from the original bias function, the original test information function and the first and second derivatives of θ with respect to τ .

7.2 Scale Transformation to Generate a Constant Test Information

In a nonparametric approach for estimating the operating characteristics of discrete item responses, the latent trait θ is transformed to τ in such a way that the resultant test information function $I^*(\tau)$ assumes a constant value for the interval of τ of interest, in which the ability levels of most subjects are included (cf. Samejima, 1981). In this way, the approximation of the conditional distribution of the maximum likelihood estimate $\hat{\tau}$, given τ , by a normal distribution with the parameters τ and σ , the latter of which does not depend upon τ , becomes more justifiable. Thus we can write

(7.9)
$$I^*(\tau) = C^2 \ , \quad C > 0 \ .$$

Substituting this into (7.6) and rearranging, we obtain for the first derivative of θ

(7.10)
$$\frac{d\theta}{d\tau} = C[I(\theta)]^{-1/2} \ .$$

The transformation of the latent trait θ to τ is given by

(7.11)
$$\tau = \tau(\theta) = C^{-1}\int_{-\infty}^{\theta} [I(t)]^{1/2}\, dt + \delta,$$

where δ indicates a constant which determines the origin of τ . From (7.10) we have for the second derivative of θ with respect to τ

(7.12)
$$\frac{d^2\theta}{d\tau^2} = -\frac{1}{2}C^2[I(\theta)]^{-2}I'(\theta) \ ,$$

where $I'(\theta)$ indicates the first derivative of the test information function $I(\theta)$ with respect to θ , for which we can write

(7.13)
$$I'(\theta) = \sum_{g=1}^{n} \sum_{k_g} P_{k_g}'(\theta)[2P_{k_g}(\theta)P_{k_g}''(\theta) - \{P_{k_g}'(\theta)\}^2][P_{k_g}(\theta)]^{-2} \ .$$

Substituting (7.12) and (7.13) into (7.8) and rearranging, we obtain

$$B^*(\tau) = B(\theta)C^{-1}[I(\theta)]^{1/2} + \frac{1}{4}C^{-1}[I(\theta)]^{-3/2}I'(\theta) \quad . \tag{7.14}$$

Thus in this specific situation (7.8) is simplified to include only the original bias function, the original test information function and its derivative, and the constant square root of test information after the latent variable has been transformed.

Let $K(\theta)$ denote the square root of the test information function $I(\theta)$. Thus we can write

$$K(\theta) = I(\theta)^{1/2} \quad . \tag{7.15}$$

Since we have

$$I'(\theta) = 2K(\theta)K'(\theta) \quad , \tag{7.16}$$

where $K'(\theta)$ is the derivative of $K(\theta)$ with respect to θ, we can rewrite (7.14) in the form

$$B^*(\tau) = B(\theta)C^{-1}K(\theta) + \frac{1}{2}C^{-1}[K(\theta)]^{-2}K'(\theta) \quad . \tag{7.17}$$

We notice from (7.17) that, in a typical situation where the square root of test information is unimodal, as is exemplified by those functions obtained for the Iowa Level 11 Vocabulary Subtest and Shiba's Word/Phrase Comprehension Test J1, which are shown in Figure 3-1, the amount of bias is decreased by the transformation of θ to τ for extreme values of the transformed latent variable where there used to be substantial amounts of bias either in negative or positive direction (cf. Figure 3-2). If we set the value of C in such a way that, for a meaningful interval of θ, the values of the two endpoints are practically unchanged over the transformation from θ to τ in order to avoid overall radical changes of scale values, then the factor $C^{-1}K(\theta)$ by which the original bias function $B(\theta)$ is multiplied, takes on values less than unity for extreme values of τ, causing reduction in the amount of bias. In addition to this fact, the second term of the right hand side of (7.17) further reduces negative biases as we go toward extremely lower levels of the transformed scale and decreases positive biases as we go toward the other extreme, for $K'(\theta)$ is positive on lower levels of τ and negative on higher levels, respectively, and $K(\theta)$ assumes small positive values on both.

Figure 7-1 presents the constant square root C of test information $I^*(\tau)$ by dashed lines, in comparison with the square root of the original test information function $I(\theta)$ which is drawn by a solid line, of the Iowa Level 11 Vocabulary Subtest. The original square root of the test information function is based upon the normal ogive model and has already been shown in Figure 3-1. The values of C and δ in (7.12) were chosen in such a way that τ is set equal to θ at $\theta = \pm 4.0$. In this way, we can avoid radical changes between the two sets of scale values. As the result, the constant square root of test information C turned out to be approximately 2.22617674. We can see in Figure 7-1 that for the interval, $(-4.0, 4.0)$, the areas under the two square roots of test information are equal. The resulting bias function $B^*(\tau)$ is shown in Figure 7-2 by a dashed line, in comparison with the original bias function $B(\theta)$. We can see a substantial decrease in the amount of bias caused by the scale transformation.

Figures 7-3 and 7-4 present the corresponding results for Shiba's Word/Phrase Comprehension Test J1. The transformation of θ to τ was made following the same strategy that was used for the Iowa Subtest. The resultant constant square root of test information C is approximately 2.39633860. We can see in this result that, after the scale transformation, the maximum likelihood estimate of τ is practically unbiased, if we accept the criterion of ± 0.1 as we did before.

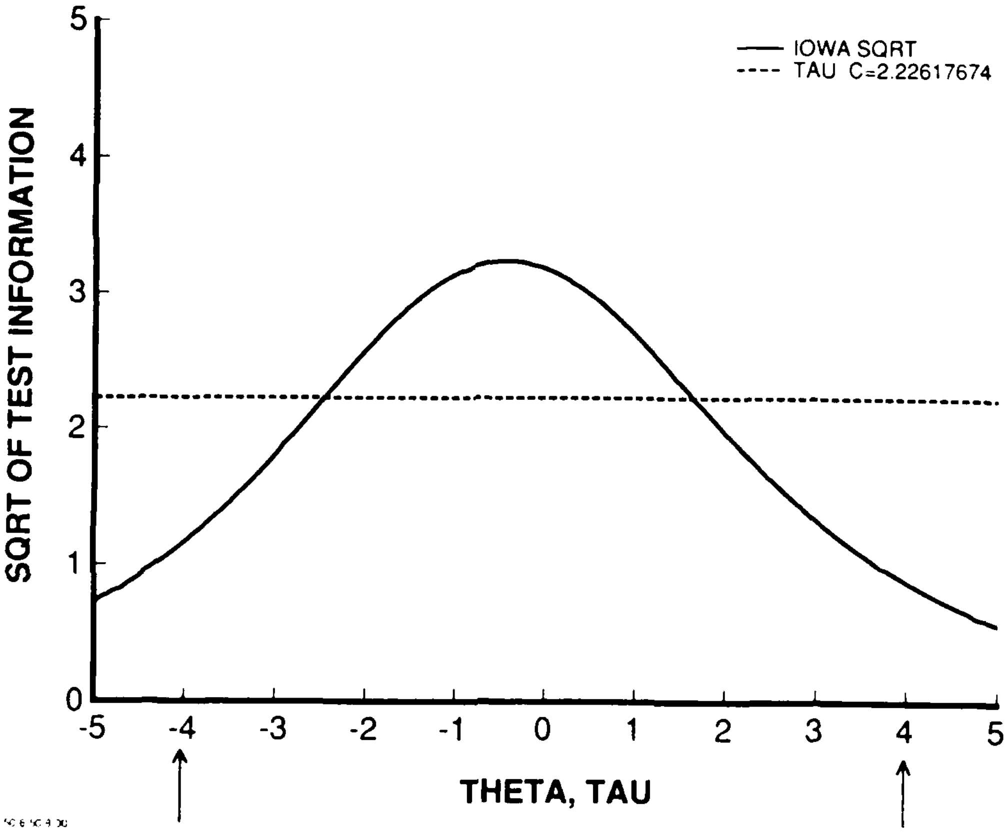


FIGURE 7-1

Square Roots of Test Information of the Iowa Level 11 Vocabulary Subtest Before (Solid Line) and After (Dashed Line) the Scale Transformation. Transformation is made in such a way that the two scales match at -4.0 and $+4.0$.

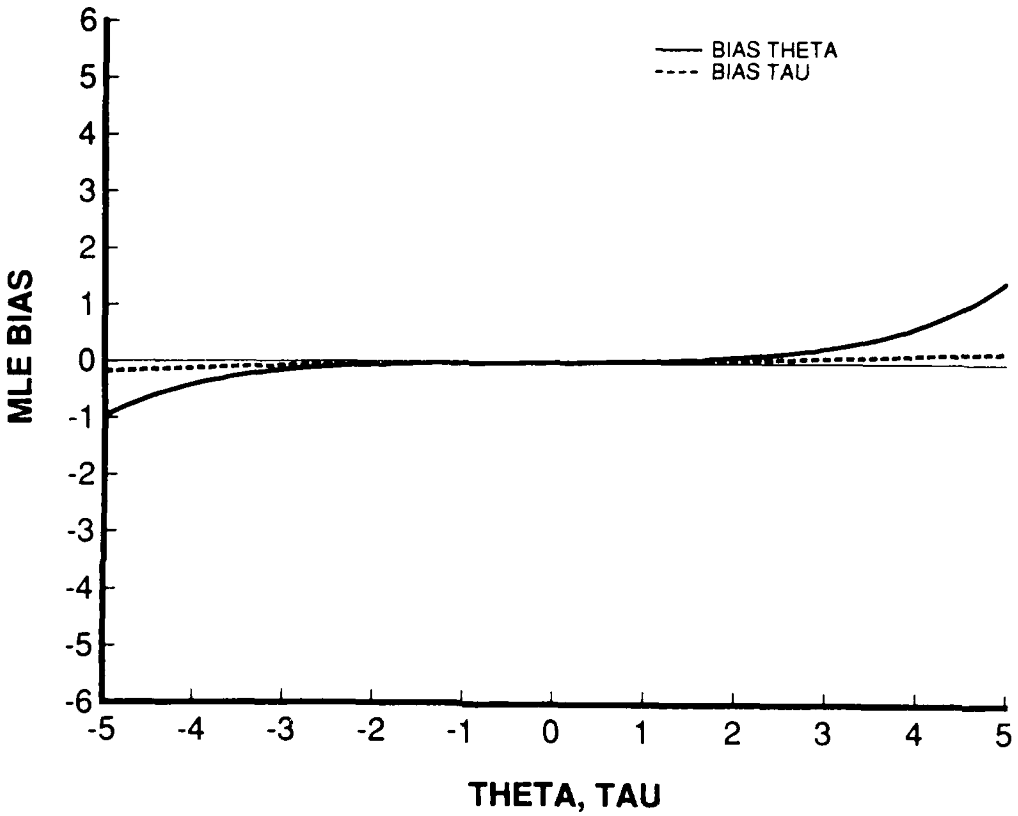


FIGURE 7-2

MLE Bias Function of the Iowa Level 11 Vocabulary Subtest as a Function of the Transformed Latent Variable τ (Dashed Line) in Comparison to the Original MLE Bias Function (Solid Line) of θ .

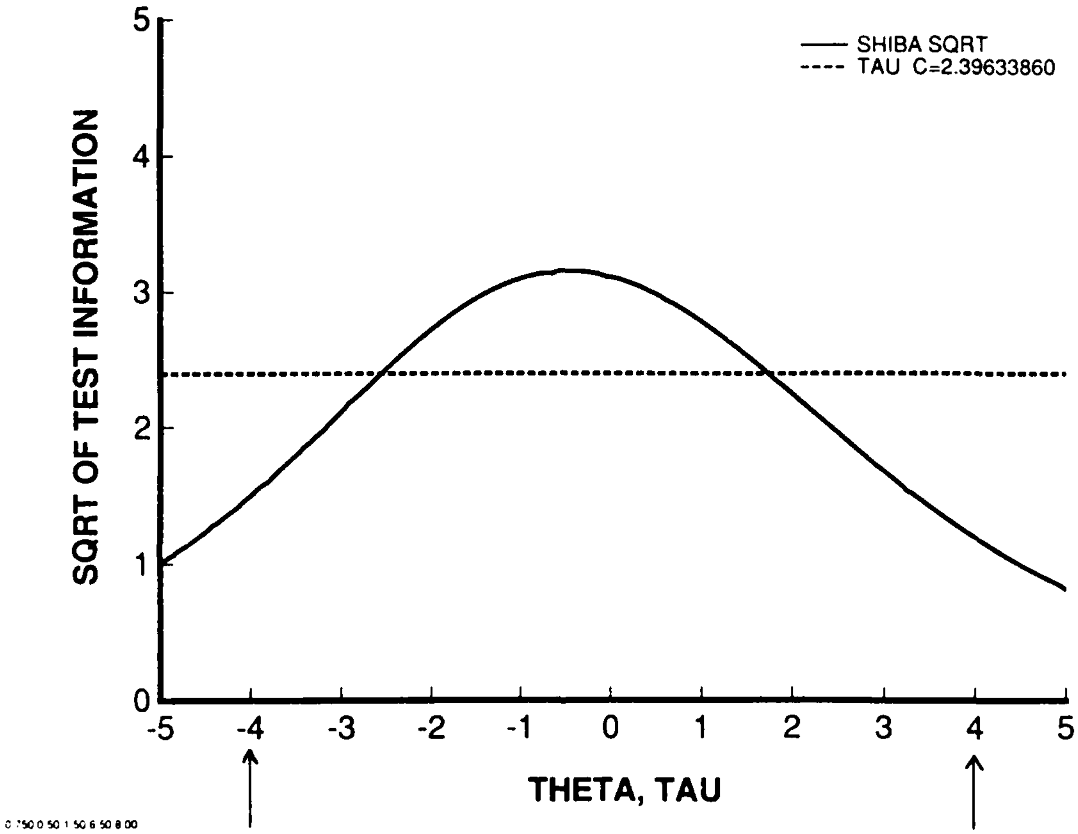


FIGURE 7-3

Square Roots of Test Information of Shiba's Word/Phrase Comprehension Test J1 Before (Solid Line) and After (Dashed Line) the Scale Transformation. Transformation is made in such a way that the two scales match at -4.0 and $+4.0$.

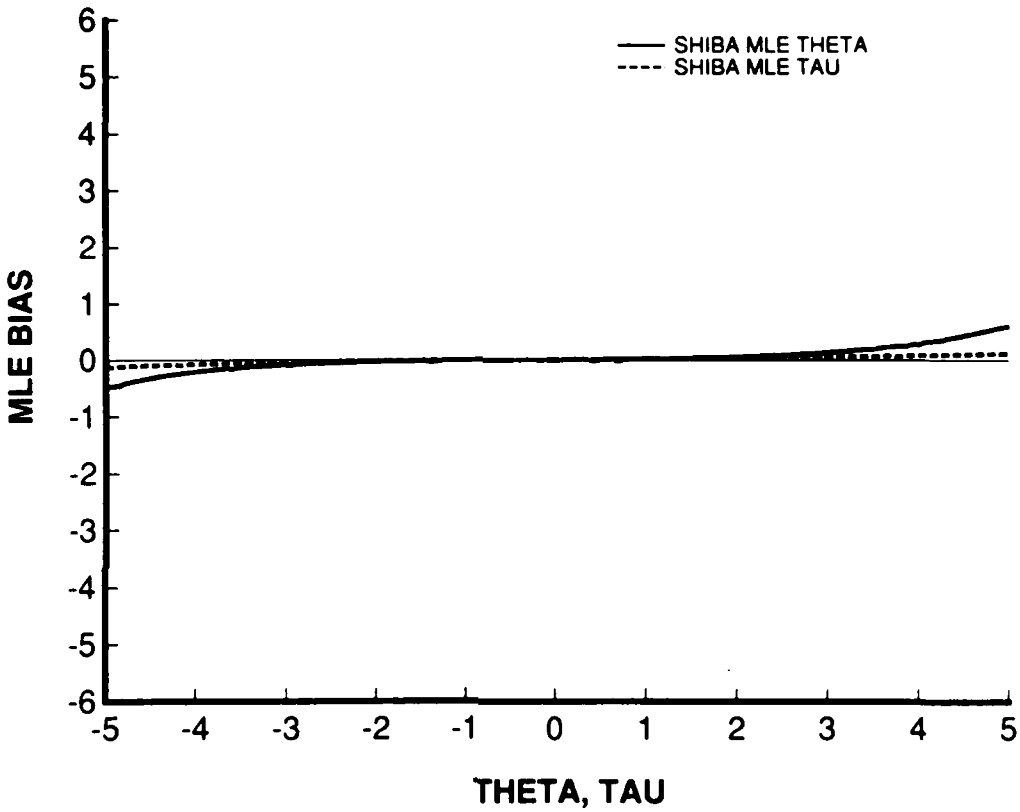


FIGURE 7-4

MLE Bias Function of Shiba's Word/Phrase Comprehension Test J1 as a Function of the Transformed Latent Variable τ (Dashed Line) in Comparison to the Original MLE Bias Function (Solid Line) of θ . Transformation of θ to τ Is Made by a Polynomial Approximation.

It has been demonstrated that the square root of the test information can be approximated by a polynomial of a suitable degree obtained by the method of moments, which proves to be also the least square solution (cf. Samejima and Livingston, 1979). It has also been shown that with many different sets of data such approximations have worked well (e.g. Samejima, 1981, 1984a). For the purpose of illustration, the polynomial approximation was used with the Iowa Level 11 Vocabulary Subtest, and the resulting scale transformation is given by

$$\tau = 0.3777014 + 1.43011120\theta - 0.05288854\theta^2 - 0.04080096\theta^3 + 0.00294040\theta^4 + 0.00110037\theta^5 - 0.00008858\theta^6 - 0.00001460\theta^7 + 0.00000010\theta^8 \tag{7.18}$$

In this scale transformation, the same strategy was taken as before, so that $\tau(\theta) = \theta$ at $\theta = \pm 4.0$. The constant square root of the test information function of τ turned out to be 2.231709 , which is very close to the corresponding value of 2.22617674, which was obtained without the polynomial approximation. The bias function $B^*(\tau)$ thus obtained is shown in Figure 7-5 by a dashed line, in comparison with the original $B(\theta)$ which is drawn by a solid line. We can see that this result is practically identical with the one obtained without the polynomial approximation, which is shown in Figure 7-2.

Figure 7-6 presents the three separate scale transformations of the Iowa Level 11 Vocabulary Subtest, of Shiba's Test J1 and of the Iowa Subtest with the polynomial approximation, by solid, dashed and dotted lines, respectively. Actually, we can only see two curves, for the dotted curve practically coincides with the solid curve.

7.3 Equivalent Items on the Dichotomous Response Level

We have seen in a previous section how the amount of bias decreases as the number of items increases, using the example of equivalent items on the dichotomous response level, which follow the constant information model. It should be noted that the corresponding set of bias functions for equivalent items following *any* mathematical model, which provides us with a strictly increasing item characteristic function with zero and unity as its two asymptotes, can be produced from these results by an appropriate strictly increasing scale transformation. Let $\tau = \tau(\theta)$ be such a transformation of θ , and $P_g^*(\tau)$ denote the item characteristic function following one of such models. Setting

$$P_g^*(\tau) = P_g(\theta) \quad , \tag{7.19}$$

we obtain

$$P_g^{*\prime}(\tau) = P_g'(\theta)\frac{d\theta}{d\tau} = 2\alpha_g\{P_g(\theta)Q_g(\theta)\}^{1/2}\frac{d\theta}{d\tau} \quad , \tag{7.20}$$

$$\begin{aligned} P_g^{*\prime\prime}(\tau) &= P_g''(\theta)\{\frac{d\theta}{d\tau}\}^2 + P_g'(\theta)\frac{d^2\theta}{d\tau^2} \\ &= 2\alpha_g[\alpha_g\{Q_g(\theta) - P_g(\theta)\}\{\frac{d\theta}{d\tau}\}^2 + \{P_g(\theta)Q_g(\theta)\}^{1/2}\frac{d^2\theta}{d\tau^2}] \end{aligned} \tag{7.21}$$

and

$$I_g^*(\tau) = I_g(\theta)\{\frac{d\theta}{d\tau}\}^2 = 4\alpha_g^2\{\frac{d\theta}{d\tau}\}^2 \quad , \tag{7.22}$$

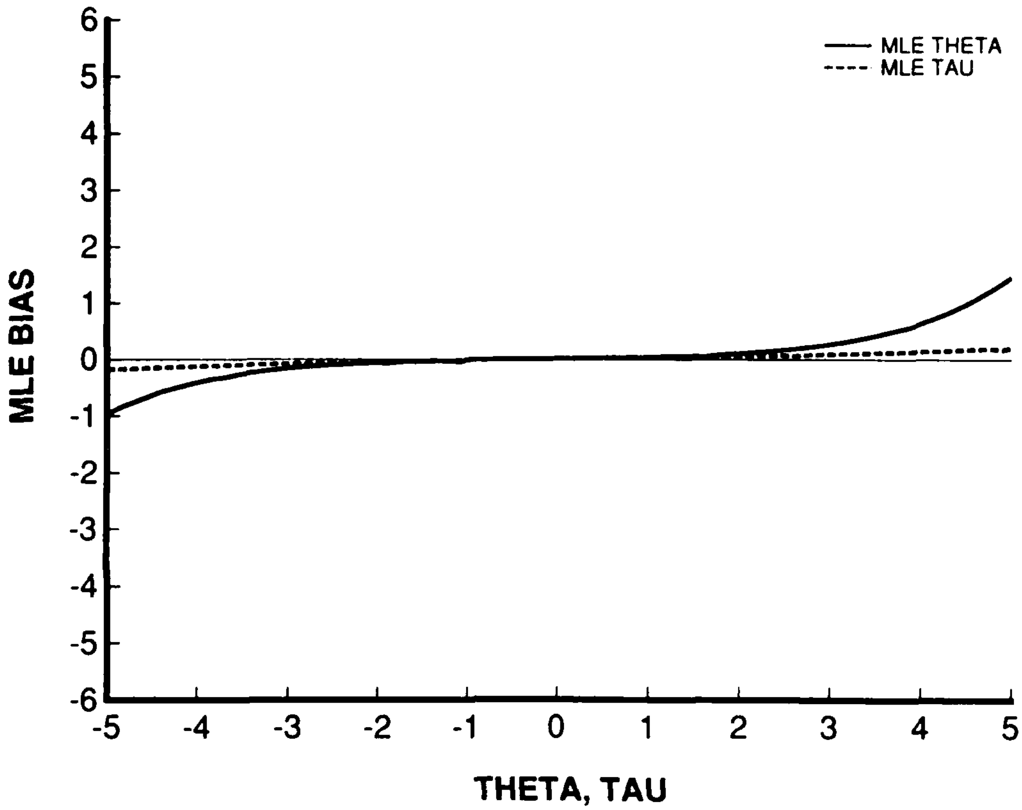


FIGURE 7-5

MLE Bias Function of the Iowa Level 11 Vocabulary Subtest as a Function of the Transformed Latent Variable τ (Dashed Line) in Comparison to the Original MLE Bias Function (Solid Line) of θ. Transformation of θ to τ Is Made by a Polynomial Approximation.

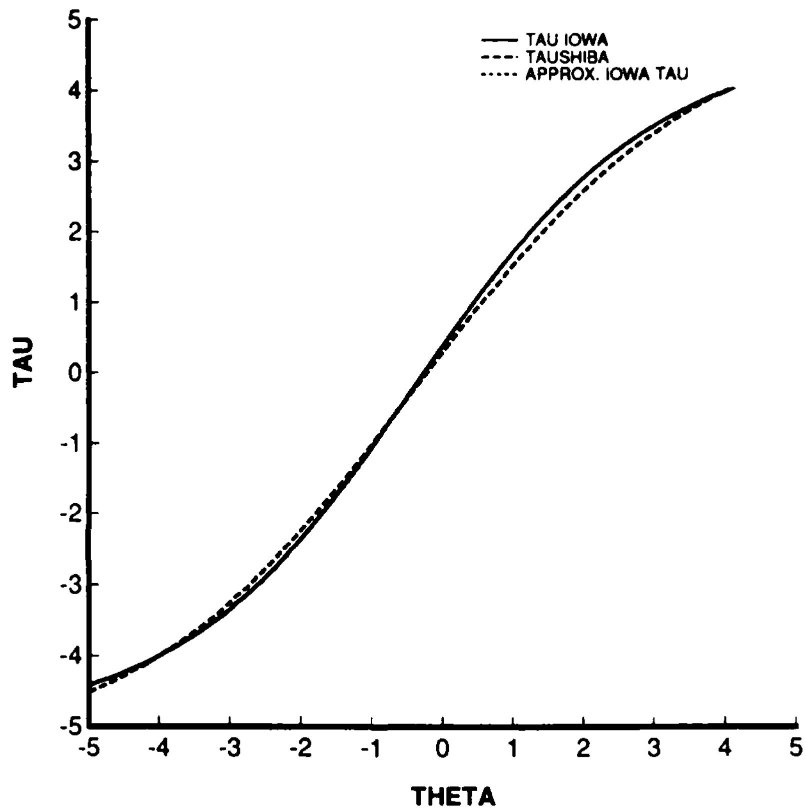


FIGURE 7-6

Transformation of θ to τ Based Upon The Iowa Level 11 Vocabulary Subtest (Solid Line), Upon Shiba's Word/Phrase Comprehension Test J1 (Dashed Line), and Upon the Iowa Level 11 Vocabulary Subtest Using the Polynomial Approximation.

where $P_g^*(\tau)$ and $I_g^*(\tau)$ indicate the item characteristic function and the item information function, respectively, after the scale transformation, and $P_g^{*\prime}(\tau)$ and $P_g^{*\prime\prime}(\tau)$ denote the first and second derivatives of $P_g^*(\tau)$ with respect to τ . Since we can write for a set of n equivalent items

(7.23) $$B^*(\tau) = (-2n)^{-1} P_g^*(\tau) Q_g^*(\tau) P_g^{*\prime\prime}(\tau)\{P_g^{*\prime\prime\prime}(\tau)\}^{-3} \quad ,$$

where $B^*(\tau)$ is the bias function after the scale transformation, we obtain

(7.24) $$B^*(\tau) = B(\theta)\{\frac{d\theta}{d\tau}\}^{-1} - (8n\alpha_g^2)^{-1}\{\frac{d\theta}{d\tau}\}^{-3}\frac{d^2\theta}{d\tau^2} \quad .$$

For the purpose of illustration, let us consider the scale transformation which changes the constant information model to the logistic model. Thus we have

(7.25) $$P_g^*(\tau) = \{1 + e^{-Da_g(\tau - b_g)}\}^{-1} \quad .$$

The functional relationship between θ and τ is given by

(7.26) $$\tau = (Da_g)^{-1}\log[\tan^2\{\alpha_g(\theta - \beta_g) + (\pi/4)\}] + b_g \quad ,$$

or

(7.27) $$\theta = \alpha_g^{-1}[\tan^{-1}\{e^{(1/2)Da_g(\tau - b_g)}\} - (\pi/4)] + \beta_g \quad .$$

The first and second derivatives of θ with respect to τ are given by

(7.28) $$\frac{d\theta}{d\tau} = Da_g(2\alpha_g)^{-1}\{P_g(\theta)Q_g(\theta)\}^{1/2}$$

and

(7.29) $$\frac{d^2\theta}{d\tau^2} = D^2a_g^2(4\alpha_g)^{-1}\{P_g(\theta)Q_g(\theta)\}^{1/2}\{Q_g(\theta) - P_g(\theta)\} \quad ,$$

respectively. Thus we can write from (7.19), (7.20), (7.21), (7.22), (7.28), and (7.29)

(7.30) $$P_g^{*\prime}(\tau) = Da_gP_g(\theta)Q_g(\theta) = Da_gP_g^*(\tau)Q_g^*(\tau) \quad ,$$

(7.31) $$\begin{aligned} P_g^{*\prime\prime}(\tau) &= D^2a_g^2P_g(\theta)Q_g(\theta)\{Q_g(\theta) - P_g(\theta)\} \\ &= D^2a_g^2P_g^*(\tau)Q_g^*(\tau)\{Q_g^*(\tau) - P_g^*(\tau)\} \end{aligned}$$

and

(7.32) $$I_g^*(\tau) = D^2a_g^2P_g(\theta)Q_g(\theta) = D^2a_g^2P_g^*(\tau)Q_g^*(\tau) \quad ,$$

where

$$Q_g^*(\tau) = 1 - P_g^*(\tau) \quad . \tag{7.33}$$

We can easily see that these results are agreeable with those obtained directly from (7.25). For the bias function $B^*(\tau)$, we have from (7.24), (7.28), and (7.29)

$$\begin{aligned} B^*(\tau) &= (2nDa_g)^{-1}\{P_g(\theta) - Q_g(\theta)\}\{P_g(\theta)Q_g(\theta)\}^{-1} \\ &= (nDa_g)^{-1}\{P_g^*(\tau)Q_g^*(\tau)\}^{-1}\{P_g^*(\tau) - (1/2)\} \quad . \end{aligned} \tag{7.34}$$

We can see that (7.34) is a special case of (1.4) when all the n items are equivalent, by replacing θ by τ and $\Psi_g(\theta)$ by $P_g^*(\tau)$.

8 Adaptive Testing

Observations made in previous sections provide us with ideas how things go in adaptive testing. First of all, in order to reach the practical unbiasedness in estimating the individual subject's ability in adaptive testing, we need to make sure that a sufficient amount of test information has been reached for each individual subject, before terminating the presentation of new items. We can control it easily, if we use the amount of test information as the criterion for the termination of presenting new items, or the stopping rule. If the items follow the normal ogive or logistic model in the adaptive testing situation, for subjects of intermediate ability levels it is likely that on the initial stage the item difficulty parameters fluctuate both negatively and positively around the subject's true ability level, and consequently, the biases of negative and positive directions are cancelled out, since an item pool usually has plenty of items of intermediate difficulties. In such a case, we do not have to worry too much about the influence of initial items on the eventual bias of the ability estimate. When the maximum likelihood estimate has started being more or less stabilized, chances are slim that the additional item causes a substantial bias, provided that the program is written in such a way that an item of a large amount of information at the current estimated ability level will be presented next, and that the item pool has a sufficient number of items whose difficulty levels are around the subject's true ability level. There is a greater possibility that the examinee obtains a biased ability estimate if his ability level is close to either end of the configuraiton of difficulty parameters, since biases caused by the initially presented items are not likely to cancel themselves out, and, moreover, there may not be a sufficient number of items whose difficulty levels are close to his true ability level.

If the item pool consists of items following the three-parameter normal ogive or logistic model, the effect of random guessing on the amount of bias can be substantial, especially on the lower levels of ability. In such a case, it is imperative to include many easy items in the item pool.

In any case, the bias function can be a good indicator in evaluating the item pool, if we use it wisely and effectively. Those results that were described in previous sections will give us information and suggestions as to how to improve an existing item pool.

9 Discussion and Conclusions

The bias function of the maximum likelihood estimate has been proposed for the general discrete response level, which includes Lord's bias function in the three-parameter logistic model as a special case. The function has also been observed both on the dichotomous and graded response levels, with respect to various mathematical models. Effects of the item discrimination parameters, of the item difficulty parameters, and of the number of items have also been observed. Local changes in the amount of bias caused by the scale transformation have also been observed from various different angles, and it has also been discussed in the context of adaptive testing.

Since the local unbiasedness is important, the proposed function will find its usefulness directly in the estimation of the subject's latent trait. An even greater usefulness of the function can be seen in the context of more elaborated methodologies, as is exemplified in the nonparametric approach to the estimation of the operating characteristics of discrete responses.

References

[1] Lord, F. M. Unbiased estimators of ability parameters, of their variance, and of their parallel-forms reliability. *Psychometrika,* 48, 1983, 233-245.

[2] Samejima, F. Estimation of latent ability using a response pattern of graded scores. *Psychometrika Monograph,* No. 17, 1969.

[3] Samejima, F. A general model for free-response data. *Psychometrika Monograph,* No. 18, 1972.

[4] Samejima, F. A method of estimating item characteristic functions using the maximum likelihood estimate of ability. *Psychometrika,* 42, 1977, 163-191.

[5] Samejima, F. Constant Information Model: A New, Promising Item Characteristic Function. *ONR Research Report,* 79-1, 1979.

[6] Samejima, F. and Livingston, P.S. Method of moments as the least squares solution for fitting a polynomial. *ONR Research Report,* 79-2, 1979.

[7] Samejima, F. Final Report: Efficient methods of estimating the operating characteristics of item response categories and challenge to a new model for the multiple-choice item. *Final Report of N00014-77-C-0360,* Office of Naval Research, 1981.

[8] Samejima, F. Plausibility Functions of Iowa Vocabulary Test Items Estimated by the Simple Sum Procedure of the Conditional P.D.F. Approach. *ONR Research Report,* 84-1, 1984.

[9] Samejima, F. Comparison of the Estimated Item Parameters of Shiba's Word/Phrase Comprehension Tests Obtained by Logist 5 and Those by the Tetrachoric Method. *ONR Research Report,* 84-2, 1984.

[10] Shiba, S. Construction of a Scale for Aquisition of Word Meanings. *Bulletin of Faculty of Education, University of Tokyo,* 17, 1978. (in Japanese)

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June 16, 1987

Dr. Terry Ackerman
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
Naval Training Systems Center
Orlando, FL 32813

Dr. James Algina
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N711
Naval Training Systems Center
Orlando, FL 32813

Dr. Bruce Bloxom
Defense Manpower Data Center
550 Camino El Estero,
Suite 200
Monterey, CA 93943-3231

Dr. R. Darrell Bock
University of Chicago
NORC
6030 South Ellis
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code N-095R
Naval Training Systems Center
Orlando, FL 32813

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. Lyle D. Broemeling
ONR Code 1111SP
800 North Quincy Street
Arlington, VA 22217

Mr. James W. Carey
Commandant (G-PTE)
U.S. Coast Guard
2100 Second Street, S.W.
Washington, DC 20593

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
OP 0187
Washington, DC 20370

Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans Crombag
University of Leyden
Education Research Center
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

Mr. Timothy Davey
University of Illinois
Educational Psychology
Urbana, IL 61801

Dr. Doug Davis
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Foundation
800 Roosevelt Road
Building C, Suite 206
Glen Ellyn, IL 60137

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Pat Federico
Code 511
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dipl. Pad. Michael W. Habon
Universitat Dusseldorf
Erziehungswissenschaftliches
Universitatsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ronald K. Hambleton
Prof. of Education & Psychology
University of Massachusetts
at Amherst
Hills House
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Mr. Dick Hoshaw
OP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Department of Psychology
University of South Carolina
Columbia, SC 29208

Dr. Dennis E. Jennings
Department of Statistics
University of Illinois
1409 West Green Street
Urbana, IL 61801

Dr. Douglas H. Jones
Thatcher Jones Associates
P.O. Box 6640
10 Trafalgar Court
Lawrenceville, NJ 08648

Dr. Milton S. Katz
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
University of Texas-Austin
Measurement and Evaluation
Center
Austin, TX 78703

Dr. James Krantz
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

Dr. Milton Maier
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William L. Maloy
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Dr. James McBride
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Clarence McCormick
HQ, MEPCOM
MEPCT-P
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Barbara Means
Human Resources
Research Organization
1100 South Washington
Alexandria, VA 22314

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters, Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

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Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

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Office of Naval Research,
Code 125
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Arlington, VA 22217-5000

Assistant for MPT Research,
Development and Studies
OP 01B7
Washington, DC 20370

Dr. Judith Orasanu
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Administrative Sciences Department,
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
Knoxville, TN 37916

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Richard E. Snow
Department of Psychology
Stanford University
Stanford, CA 94306

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

Maj. Bill Strickland
AF/MPXOA
4E168 Pentagon
Washington, DC 20330

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL
Bolling AFB, DC 20332

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 310
St. Paul, MN 55114

Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
Program Manager
Manpower Analysis Program
HumRRO
1100 S. Washington St.
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

Dr. Ronald A. Weitzman
NPS, Code 54Wz
Monterey, CA 92152-6800

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90007

German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152-6800

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
Memory & Cognitive
Processes
National Science Foundation
Washington, DC 20550

NON GOVT

1 Dr. Robert Guion
Department of Psychology
Bowling Green State University
Bowling Green, OH 43403

1 Dr. William W. Turnbull
Educational Testing Service
Princeton, NJ 08542

1 Dr. P. Mengal
Faculte de Psychologie
et des Sciences de l'Education
Universite de Geneve
3 fl. de l'Universite
1201 Geneva SWITZERLAND

1 Dr. Wim J. van der Linden
Vakgroep Onderwisjskunde
Postbus 217
7500 EA Endschede
THE NETHERLANDS

1 Dr. Lutz Hornke
University Duesseldorf
Erz. Wiss.
D-4000 Duesseldorf
WEST GERMANY

1 Dr. Wolfgang Buchtala
8346 Simbach Inn
Postfach 1306
Industriestasse 1
WEST GERMANY

1 Dr. Albert Beaton
Educational Testing Service
Princeton, NJ 08542

1 Dr. Sukeyori Shiba
Faculty of Education
University of Tokyo
Hongo, Bumkyoku
Tokyo, JAPAN 113

NON GOVT

1 Dr. Takahiro Sato
Nippon Eletric Co., Ltd.
C & C Systems Research Laboratories
4-1-1 Miyazaki
Miyamaeku, Kawasaki
Kanagawaken 213, JAPAN

1 Dr. J. Uhlaner
Perceptronics, Inc.
6271 Variel Ave.
Woodland Hills, CA 91634

1 Mr. Susumu Fujimori
1-21-36 Hiyoshi
Kohokuku, Yokohama 223
JAPAN

1 Mr. Kenji Goto
Ina Nyuhaim #208
7-2-12 Honcho, Tanashi-shi
Tokyo 188, JAPAN

1 Dr. G. Gage Kingsbury
Portland Public Schools
Evaluation Department
501 North Dixon Street
Portland, Oregon 97227

1 Mr. Tadashi Shibayama
Ono-kohpo #201
901 Hiregasaki
Nagareyama-shi
Chiba-ken, 270-01
JAPAN

1 Dr. Mark Wilson
School of Education
University of California
Berkeley, CA 94720

1 Dr. Chang-I Bonnie Chen
Graduate School of Psychology
National Chenglhi University
Taipei, Taiwan
R.O.C.

NON GOVT

1 Dr. Ivo W. Molenaar
F.S.W.-R.U.G.
Oude Boteringestraat 23
9712 GC Groningen
THE NETHERLANDS

NAVY

1 Mr. Thomas Bryant
Office of Naval Research
206 O'Keefe Building
Atlanta, GA 30332

ARMY

1 Dr. Randall M. Chambers
U.S. Army Research Institute for
the Behavioral & Social Sciences
Fort Sill Field Unit
P.O. Box 3066
Fort Sill, OK 73503

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